

On Robustness of Three Linear Time Series Models to an Outlier

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Abstract

This article compared the robustness of decomposition, Holt-Winters and SARIMA models to an outlier. Data are average monthly minimum and average monthly maximum temperature data for Bida, collected from National Cereals Research Institute, Badeggi, Bida, Nigeria. Original series and series with an outlier were analyzed via the three models in contention. Mean absolute percentage error was the performance criterion used. Out-of-sample forecast accuracy was used as a proxy for robustness. Results revealed Holt-Winters and decomposition as the most robust for minimum and maximum temperature data respectively. SARIMA proved to be least robust for both series. For the minimum temperature series, decomposition followed Holt-Winters while for the maximum temperature series, Holt-Winters trailed behind decomposition. More series may be required to know the more robust of decomposition and Holt-Winters. It was suggested that more series be included in future studies.

Keywords: Time Series, Outlier, Robustness, SARIMA, Holt-Winters.

Introduction

In various areas of human endeavor, large amounts of data are generated which are analyzed to aid decision making process. It is logical that results, on which decisions are made, be it at the level of the individual or corporate must be of high quality so that the decision maker is not misled. Apart from the need to apply appropriate tools, the data must also conform to model assumptions appreciably. A number of issues can adversely affect results (parameter estimates) obtained from modeling. One important aspect of such is the presence of outliers.

An outlier, or anomaly, is an observation that deviates from others so greatly that it raises suspicion in the dataset (Hawkins, 1980). Imagine the appearance of 850° C in a series on atmospheric daily maximum temperature! This calls for investigation as it is not realistic. An outlier is not necessarily a nuisance observation. Appearance of outliers can provide an insight into the time series; the appearance of an outlier may also be an indication of a new trend in series generating process.

Osborne and Overbay (2004) briefly categorized the negative effects of outliers on statistical analyses: Outliers generally serve to increase error variance and reduce the power of statistical tests; if non-randomly distributed, they can decrease normality (and in multivariate analyses, violate assumptions of sphericity and multivariate normality), altering the odds of making both Type I and Type II errors; they can seriously bias or

influence estimates that may be of substantive interest. Outliers can lead to model misspecification, biased parameter estimation and poor forecasts.

A considerable amount of research works has been published on outlier. While some of these works dwelt on effect of outlier(s) on specific models, others dwelt on algorithms for detecting outliers. Ledolter (1989) studied the effect of additive outliers on forecasts from ARMA model. It was found that forecast intervals are quite sensitive to additive outliers. Martin and Yohai (1985) examined robustness of ARMA estimation. Chang, Tiao and Chen (1988) studied on estimation of time series parameters in the presence of outliers. Deutsch, Richards and Swain (1990) studied the effects of a single outlier on ARMA identification. More recent studies on outlier dwelt on outlier detection methods rather effects on model forecasting performance. Such studies include Osborne and Overbay (2004), Louni (2008), Choi, Um and Choa (2016), Battaglia and Cucina (2020). Others are Vishwakarma, Paul and Elsayah (2020), Lee, Jeong, Joo, Yoon, Han and Baik (2022), Benko, Babel, and Somogyvari (2022).

For univariate data, the concept of outlier seems relatively simple to define. Plots such as scatter plot, QQ-plot and stem-and-leaf plot can often reveal outlying observations. This does not hold for multivariate outliers as visual methods simply do not work. Multivariate outliers pose challenges that do not occur with univariate data. The concern of this research is however, univariate outlier.

Statistical model estimates are generally known to be distorted by appearance of outlier(s); time series models are not exempted from such distortions. The robustness of parameter estimates to appearance of outlier(s) should not be expected to be same across models. Research efforts are required to appreciate degrees of robustness of various models to appearance of outlier(s). This research is one such effort and aims to compare robustness of three linear univariate time series models (decomposition, Holt-Winters and ARIMA) to an outlier. Although extensive research has been carried out on outlier, no single study exists which compared the three models for robustness to an outlier, this is the gap the research is expected to fill.

The remaining part of the article proceeds as follows: Section 2 presents the Methodology; Section 3 presents Results and Discussion and the last section concludes the article.

Methodology

Data

Data collected are: Monthly minimum and maximum temperature of Bida from January 2003 to August 2016, data from January 2003 to December 2015 were used for the analysis and the rest were used for evaluation. Data were collected from National Cereals Research Institute, Badeggi, Bida, Nigeria.

An outlier was generated by multiplying the mean of each series by 10 and was used to replace the middle observation.

Model

The linear time series models involved are classical decomposition model, Holt-Winters model and SARIMA model.

Decomposition Model

The Decomposition model of interest is the multiplicative model:

$$Y_t = T_t \cdot S_t \cdot C_t \cdot I_t \quad (1)$$

where T, S, C and I represent trend, seasonal, cyclical and irregular component respectively.

The multiplicative version of Holt-Winters model is of interest and its form follows:

$$\begin{aligned} l_t &= \alpha \frac{y_t}{s_{t-p}} + (1 - \alpha)(l_{t-1} + b_{t-1}) \\ b_t &= \beta(l_t - l_{t-1}) + (1 - \beta)b_{t-1} \\ s_t &= \delta \frac{y_t}{l_t} + (1 - \delta)s_{t-p} \\ \hat{y}_{t+k} &= (l_t + kb_t)s_{t-k-p} \end{aligned} \quad (2)$$

α, β and δ are smoothing constants accounting for the level, trend and the seasonal components respectively. l_t, b_t and s_t are smoothed estimates of the level, trend and the seasonal components at time t respectively. p is seasonality and equals 12 and 4 respectively for monthly and quarterly data. $\hat{y}_{t+k}$ is the k -step-ahead forecast.

SARIMA Model

Developed by Box and Jenkins (1976), SARIMA (p, d, q) $\times$ (P, D, Q)_s is of the form

$$\phi_p(B)\Phi_p(B^s)(1-B)^d(1-B^s)^D y_t = \theta_q(B)\Theta_Q(B^s)\varepsilon_t \quad (3)$$

where p, d, q are integers and represent the number of autoregressive parameters, the order of nonseasonal differencing and number of moving average parameters in the model respectively. P, D and Q are seasonal counterparts of p, d and q . When P, D and Q are all zero, SARIMA becomes ARIMA. This is appropriate when data is not seasonal.

Model Selection Criterion

The model selection criterion employed is the mean absolute percentage error (MAPE), defined as:

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right| \times 100\% \quad (4)$$

n = number of data points; A_t = actual value and F_t = predicted value

Results and Discussion

Figures 1 to 9 present the plots for the models under consideration for both cases of original series (series without an outlier) and series with an outlier. Figures with asterisks refer to minimum temperature series while others refer to maximum temperature.

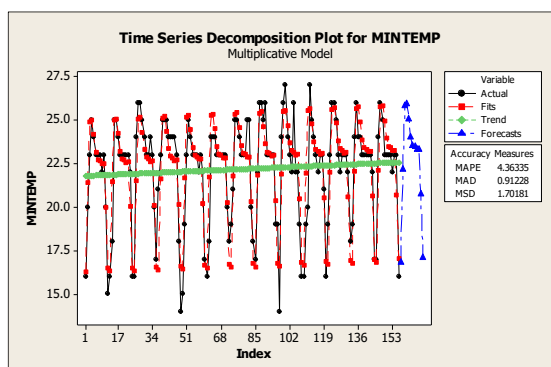


Fig. 1. Decomposition model plot without an outlier.

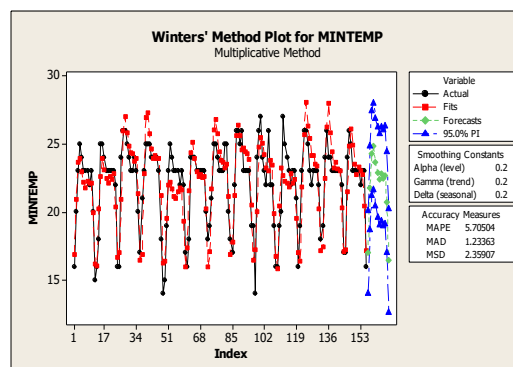


Fig. 2. Holt-Winters model plot without an outlier.

Fig. 1 portrays the minimum temperature series as having a regular periodicity. A visual inspection of the forecasts shows a forecasting behavior that mimics the original series appreciably. Forecasts obtained from Holt-Winters also portray similar behavior (see Fig. 2).

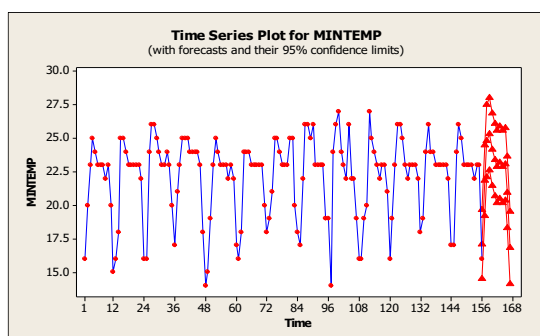


Fig. 3. SARIMA model plot without an outlier.

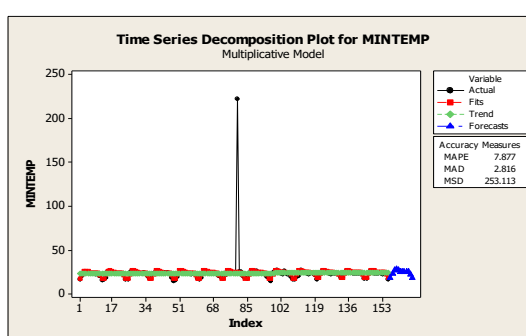


Fig. 4. Decomposition model plot with an outlier.

SARIMA forecasts seem to behave in a similar manner to decomposition and Holt-Winters models (See Fig.3)

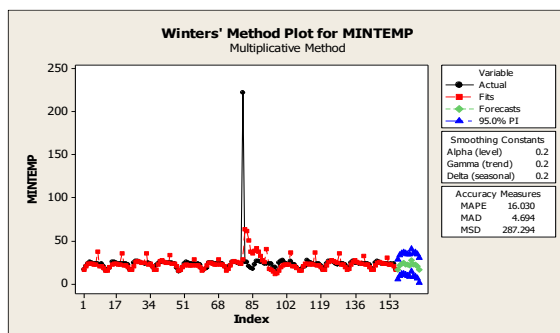


Fig. 5. Holt-Winters model plot with an outlier.

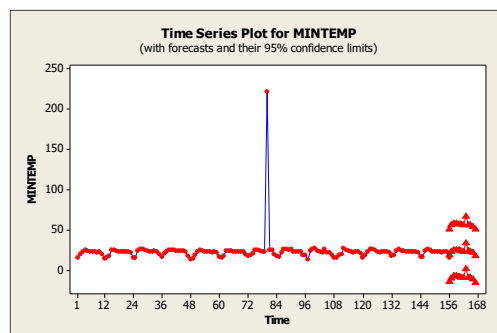


Fig. 6. SARIMA model plot with an outlier.

Figures 4, 5 and 6 show forecasts in the presence of an outlier. The forecasts seem not radically distorted from the pattern of the original series, probably because only an outlier is present.

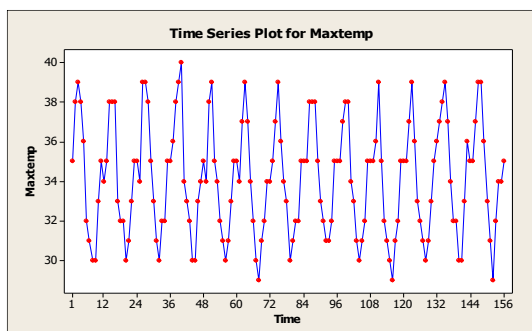


Fig. 7. Plot of max. temperature without an outlier.

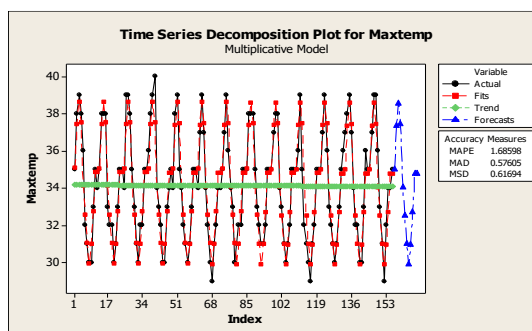


Fig. 8. Decomposition model Plot without an outlier.

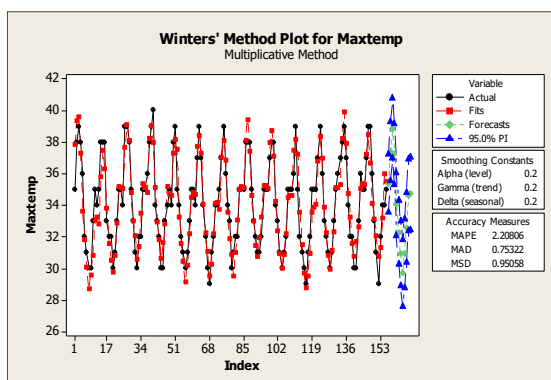


Fig. 9. Plot of H-W model without an outlier.

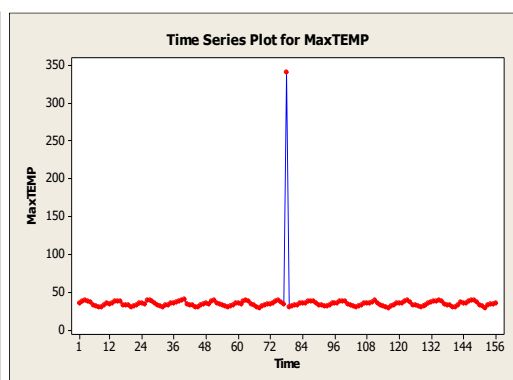


Fig. 10. Plot of max. temp. with an outlier.

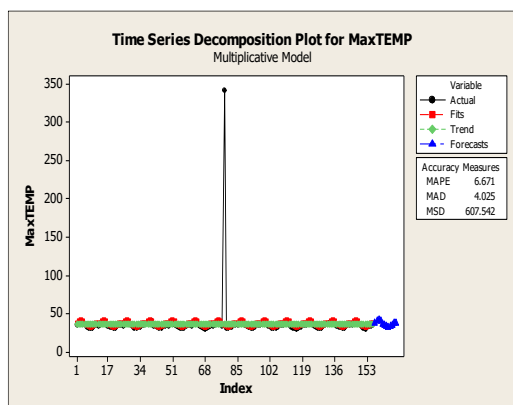


Fig. 11. Decomposition model plot with outlier.

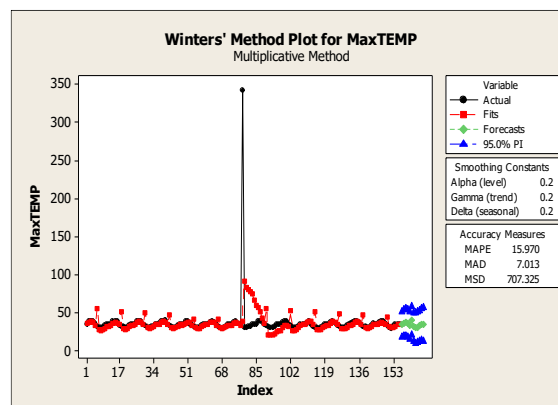


Fig. 12. H-W model plot with outlier.

Just the case of minimum temperature, forecasts obtained for both the original series and one with an outlier appear to be in harmony with pattern followed by the original series. Accuracy measures will however, be required to distinguish the performance of one model from the other.

Table 1. Forecasts and mean absolute percentage errors (Minimum temperature)

		Outlier Status					
		Without an outlier			With an outlier		
Period	Actual	Decomp	H-W	SARIMA	Decomp	H-W	SARIMA
157	16	16.8456	17.0498	17.0989	17.8549	16.5763	18.4342
158	19	22.1893	21.7971	21.8605	22.7924	21.128	22.9653
159	26	25.8099	24.3195	24.8064	27.3576	23.4008	25.7893
160	25	25.9097	24.8343	25.3309	27.4641	23.7815	25.5823
161	24	25.0336	23.6597	24.164	26.536	22.5697	24.9357
162	23	24.0011	22.9252	23.3945	25.4422	21.8214	23.9785
163	22	23.547	22.411	22.8781	24.9614	21.324	23.5938
164	22	23.4592	22.7944	23.1963	25.1261	27.1827	34.327
MAPE		6.0956	4.4539	4.9582	11.9355	8.4234	13.8316
% change in MAPE					95.81	89.12	178.96

For minimum temperature series without an outlier, Holt-Winters model performed best, followed by SARIMA and decomposition model performed worst based on MAPE (See Table 1). In the case of series with an outlier, Holt-Winters maintained the lead but followed by decomposition model and SARIMA came last. This implies that decomposition and SARIMA swapped positions, comparing performance on series with outlier to the one without.

Expectedly, the MAPE increased appreciably for each model with the introduction of the outlier. However, the incremental rate is not same for all models, hence, the swapping of

positions recorded. The decomposition model seems to be more robust to an outlier than SARIMA as far as the series under consideration is concerned since it displaced SARIMA in the presence of an outlier. Consistency in the relative performance of Holt-Winters suggests greater robustness to an outlier than the other models. SARIMA is hence, the least robust, followed by decomposition while Holt-Winters model is the most robust. A comparison of percentage change in MAPE after introduction of an outlier suggests that SARIMA with the highest percentage change is least robust, followed by decomposition and Holt-Winters with the least percentage change is the most robust. This is a further validation of the relative forecast performance claim.

Table 2. Forecasts and mean absolute percentage errors (Maximum temperature)

		Outlier Status					
Period	Actual	Without an outlier			With an outlier		
		Decomp	H-W	SARIMA	Decomp	H-W	SARIMA
157	36	34.99	35.38	35.63	36.89	34.45	39.04
158	38	37.36	37.41	37.95	39.62	36.31	41.06
159	39	38.57	38.88	39.23	40.91	37.7	42.21
160	40	37.45	37.22	37.42	39.74	36.09	40.49
161	36	34.06	34.08	34.3	35.88	33.11	37.42
162	32	32.51	32.27	32.76	34.5	39.81	52.58
163	31	30.99	30.96	31.64	32.89	31.09	34.96
164	31	29.88	29.69	30.55	31.69	29.76	33.61
MAPE		2.824391	2.633059	2.3515556	3.593904	7.323201	14.42542
% change in MAPE					27.25	178.13	513.44

For the case of maximum temperature, (See Table 2), SARIMA performed best, followed by Holt-Winters performed and then, decomposition for series without an outlier. This is contrary to relative performance recorded in the case of minimum temperature. This is only laying credence to the assertion that no model performs best on all series.

After introduction of an outlier, decomposition model performed best while SARIMA performed worst and Holt-Winters maintained its second position. SARIMA has proved to be most affected by the introduction of an outlier. Once again, SARIMA has recorded the maximum change in MAPE (suggesting least robustness), followed Holt-Winters while decomposition recorded the least, suggesting highest robustness. The position of SARIMA is in harmony with results obtained for the minimum temperature series. However, Holt-Winters and decomposition models swapped positions making it difficult to comment on which is better. This is only informing that reaction of a model to presence of an outlier depends to some extent on the series involved. Involvement of more series in future research may be of help.

Conclusion

The article has compared the robustness of decomposition, Holt-Winters and SARIMA models to an outlier based on mean absolute percentage error. It was found that for the minimum temperature series, Holt-Winters is the most robust, followed by decomposition while SARIMA is the least robust; and for the maximum temperature series, decomposition is the most robust, followed by Holt-Winters while SARIMA is the least robust. SARIMA is hence, the least robust of the three models. More series may be required to know the more robust of decomposition and Holt-Winters. SARIMA, being less robust than the other two may not be generalized, results have shown that reaction of a model to presence of outlier depends on the series. This is confirmed by swapping in relative performance of decomposition and Holt-Winters models, on comparing results from the two series analyzed. It is suggested that more series be included in future studies.

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