

A Systematic Review of Multi-Server Queueing System: A Case of Lengthy Wait Times in Hospital Medical Outpatient Department (OPD)

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Abstract

Lengthy wait times in hospital medical outpatient's department (OPD) is a disturbing phenomenon to many healthcare centers in Nigeria and across the globe, because of its numerous effects like overcrowding, patients leaving out of frustrations without receiving treatment, as well as stress on both staff and patients. Problems related to patient scheduling and queueing in medical outpatient department (OPD) are gaining increasing attention in theory, in the field of operations research/optimization and healthcare services and in practice. This paper aims to bring together an extensive review on related studies that addressing the problems of queueing systems of medical outpatient department (OPD). The method employed during the data collection were direct observation (primary data) from the staff (physicians) and the patients who visited the medical outpatient's clinic of the two hospitals. The data on the arrival time, waiting time, number of server, service time, utilization rate, etc. were collected on weekdays (Mondays through Fridays) for the period of four (4) weeks in each hospital. The research design adopted in this research is a case study survey that will examine two hospitals out of many numbers of hospital/clinic in Nasarawa State. Also, this paper aimed at/or research on the various methods of reducing or if not eliminating completely the lengthy wait times; method that ranges from increase in staff strength and also to build an alternative hospital/clinic. This research focuses on the introduction and review of previous literatures related to multi-server queueing systems, the methodologies outline for the purpose of this work are considered, findings and literature gaps in the research also considered. Based on the literature reviewed, the leading areas in medical informatics research is the electronic medical records. Therefore, from the findings of this work, it is recommended that management, policy makers at national level and other health facilities with similar queueing problem improve the overall patient care by installing the optimum number of server in order to meet the patient needs. The significance of the study is to provide sufficient information to the health service providers, government and national to improve service delivery to reduce customer mortality rate.

Keywords: Multi-Server, Lengthy Wait Time, Queue, Medical Outpatient, Healthcare Centers, Treatment.

Introduction

The study of multi-server queuing analysis to measure the performance of healthcare systems in hospitals is a topic of significant importance in healthcare management and operations research. Queueing theory is a mathematical approach used to model and analyze waiting lines, and it has been widely applied to healthcare settings to improve patient flow and reduce waiting times.

Around 1909, a Danish engineer named Erlang developed queueing theory as a result of his research work (Winston, 1991). He conducted an experiment and discovered that telephone traffic demand fluctuates. Following that, he released a report on the delays in automatic dialing equipment. Shortly after his published work and during the end of World War II. Erlang's early work was extended to more general challenges and business applications of waiting lines and to various service industries. Modeling a service industry as a queueing system has a number of benefits, including diagnosing problems and identifying restrictions in order to better understand real-world systems rather than making specific predictions about them. Queueing theory is the mathematical study of waiting lines, or the act of joining a line (queues). In queueing theory, a model is built so that queue lengths and waiting times can be calculated (Sundarapandian, 2009).

The issue of queueing has been a subject of scientific debate for there is no known society that is not confronted with the problem of queueing. Queueing situations arise in all aspects of work and life and are typified by the process of queueing for services, i.e., a set of physical units (people or things) which wait in a queue or queues subject to certain rules of behavior before some services are performed on or for each unit in the queue one after the other (Burodo, Suleiman & Shaba, 2019).

Wherever there is competition for limited resource, queueing is likely to occur (Koko, Burodo & Suleiman, 2018). Queues emerged when individuals requesting service, usually called customers, arrive at a service facility and cannot be served on time (Suleiman, Burodo & Ahmed, 2022).

Waiting time is defined as the time spent by the patient in the queue before the commencement of the services (Shanmugasundaram & Umarani, 2015).

Unfortunately, this case happens in many teaching hospitals. Teaching hospitals received a large number of patients every day and this generally results in long patient waiting time (Afrane & Appah, 2014).

Waiting for service is a common phenomenon in all setting where patients are involved, such as hospitals, gas stations, department stores, restaurants and banks (Adewole, 2016). Whenever patients arrive at a service facility, some of them have to wait before they receive the desired service. It means that the patients have to wait for his/her turn. This situation can be frustrating when patients have to wait for a long time before they are served in a queue, this leads to patient's dissatisfaction as they cannot have the desired level of service (Nkrumah Yeboah & Adiwokor, 2015).

Similarly, some of the patients die waiting for their turn of service. Service is delayed when the service demand is more than the available capacity. When there is low capacity in comparison to the demand then the queue will form in the system (Ahmed & Ali, 2021).

Statement of Research Problem

In most healthcare centers particularly in Nigeria, waiting lines have become a common occurrence and impediment to providing healthcare services (Mittal & Sharma, 2020).

In this context, understanding the performance of healthcare systems is crucial to ensure quality care, patient satisfaction, and efficient resource utilization. One of the critical aspects of healthcare system performance is the management of patient queues, where patients wait for services or treatments.

To address this problem, this study aims to investigate the use of multi-server queueing analysis to measure the performance of healthcare systems in two hospitals. The study will examine the patient flow, resource utilization, and service delivery to the patients by the hospitals using a mathematical model that considers multiple servers, service times, and arrival rates.

Objectives of the Review of Multi-Server Queue

The following are the objectives of the review.

- i. To work on the lengthy wait times in medical outpatient's unit
- ii. The methodologies adopted and are presently in place in reducing the wait times in medical outpatient department.
- iii. To come up with an informatics solution to reducing wait times in medical outpatient department.

Characteristic of a Queuing System

The basic features of a queuing system are (i) the input (arrival pattern) (ii) service mechanism (service pattern) (iii) the queue discipline (iv) customer's behavior

i. Input: It describes the way in which the customers arrive and join the system. Generally, customers arrive in a random fashion which is not worth making the prediction. Thus, the arrival pattern can be described in terms of probabilities, and consequently the probability Distribution for inter-arrival times (the between two successive arrivals) must be defined.

ii. Service Mechanism or Service Pattern: this means the arrangement of service facility (Physician) to serve customers. If there is infinite number of servers then all the customers are served instantaneously on arrivals and there will be no queue. But on the other hand, if the number of servers is finite then customers are served according to a specific order with service time a constant.

iii. Queue Discipline: It is the rule according to which the customers are selected for service when a queue has been formed.

The most common disciplines are:

- First come first served (FCFS)
- First in first out (FIFO)
- Last in first out (LIFO)
- Selection for service in random order (SIRO)

There are various other disciplines according to which a customer is served in preference over the other. They are pre-emptive and non-pre-emptive systems. In pre-emptive system the patient of high priority is serviced before the low priority customer (patients). In non-pre-emptive system, patients of low priority are considered before that of high priority.

iv. Customer Behavior: The customers (patients) generally behaved in the following ways namely.

- a. Reneging: A customer (patients) at a point of arriving to the hospital and sees that the queue is too long, and he has no time to wait or has no sufficient waiting space, then leave.
- b. Balking: it occurs when a waiting customer (patients) leaves the queue due to impatience.
- c. Jockeying: customers that jockey from one waiting line to another. It is most common in the hospitals, bank, supermarket etc.
- d. Priorities: In certain applications, some customers are served before others regardless of their arrival. These customers have priority over others.

Delays are as a result of disparity between demand for a service and capacity available to meet that demand. Usually, this miss-match is temporary and due to natural variability in the timing of demands and in the duration of time needed to provide service.

Timely access has been identified as one of the key elements of health care quality (institute of medicine 2001) and consequently decreasing delays has become a focus in many health care institutes. Queuing analysis can be extremely valuable tool in utilizing primary goals to provide a basic understanding of queuing helpful in designing and managing health care system (Agyei, 2019)

Health Care System

Health care is an act delivered by the health professionals, is the maintenance of health prevention, diagnosis, and treatment from mental and physical impairments. Healthcare systems exist to assure and help people, to maintain the optimal state. The improvement of health industry in a country is one of the indicators for economic development and priority, as it is directly associated with the value of human resources. Hospital strategy is one of important infrastructure that facilitate healthcare service. Hospital strategies differentiate between three care flows: patients, materials, and information flows. The patient flow involves the movement of patients to get access for medical care and related resources in the healthcare. The material flow includes the supply of medical pharmaceutical and surgical consumable, medical equipment, and devices necessary to

support the medical personnel and unquestionable patient at the hospitals and information flow recognizes information from clinic to clinics and around the hospitals.

A time spent at the hospital are often the unkind experience for patients and are the causes for frustration. This could arise, along with other many, from long waiting times for both scheduled as well as non-scheduled appointments, and delay in diagnosis and treatments. Waiting time in this research is the time spent by the patient waiting before he gets the desired consultation by the physicians. The best service is the one that provides quick delivery without a need to wait for long, which will incur additional costs to the hospital as it requires addition of service facilities (server) and technologies (Dachyar, Farizal & Yafi, 2018).

These long waiting times for services or/and short consultation hours in health systems can worsen the severity of disease and increase the socio-economic costs and may result to unnecessary suffering, stained relationship between the physician and the patient which leads to patient dissatisfaction (Aeenparast, Farzadi & Martin, 2010).

In developing country like Nigeria, the number of trained physicians is less than what the system demands. Thus, the physicians are subjected to overloading, attending to outpatients' multiple centres and being required to show up at the in-patients' hospitals. These result, the consultation hours to be limited for only a short period of time usually either morning or afternoons. Moreover, even in top, this short period of service, regrettably, the service often become inactive due to late arrivals of physicians and other interruption (Babes & Sarma, 1991).

Aronsky (2008), Emergency Department overcrowding is an international crisis that affects the quality of health care service. This is indeed true in that most hospitals offering Emergency services are overcrowded and many patients will not receive the service in time or may even loose life waiting.

A study conducted by Paul and Li (2008) described that a hospital's efficiency in service provision depends on the number of available staff, availability of intact medical equipment's including operating rooms, laboratory, supplies of water, power, medical gases like oxygen and the state of the building. This scenario is evident in many hospitals which do not have enough facilities to handle all the incoming traffic.

Obamiro (2010) studied the waiting line for expectant women in Ante natal care unit. The results of the study evaluated the effectiveness of a queuing model in identifying the shortcomings in the facilities that served the expectant mothers. The greatest Challenge was the waiting time of each mother to receive service depending on her time of arrival.

Schoenmeyr, et al (2009) analyzed some of the healthcare organizations functioning with very small differences, so decisions on compelling the scarce resources must be done well so that the investment will lead to the desired result. Queuing approach to waiting time problems is useful because it enables the research of future scenarios for which historical data are not applicable. Waiting times calculations assist in establishing the rate of service on hospital waiting lists and are more reliable measure of hospital performance than the

size of the waiting list. In some cases, the patient may be removed from a waiting list and the reasons may include that they no longer require the procedure, are instead admitted as an emergency patient, receive their treatment at a different hospital.

Medical Outpatient Department (OPD)

A medical outpatient department (OPD) is a section of a healthcare facility, such as a hospital or clinic, where patients receive medical consultations, evaluations, and treatments without being admitted to the hospital. The primary purpose of an OPD is to provide medical care for non-emergency cases, such as routine check-ups, follow-up appointments, and minor procedures.

Wait Times in Medical Outpatient Department

Services are rendered to clients on a rational basis of who comes in first, to avoid preferential treatment which may lead to chaos. So, the time between when a person arrives for a service and when he or she is attended to is referred to as wait time. In hospital medical outpatient department, the wait time is the time from when a patient comes in, through registration to seeing the physician, to the collection of medication at the pharmacy and to being discharged to return home. This time may however be extended further if the patient is evaluated to need admission into the in-patient ward.

Research on the causes of lengthy wait time in medical outpatient department has not been conclusive and there is also a shortage of such research as reported in literature searched. Chan et al. (2008) reported that data was collected to investigate causes of lengthy wait time using a diagnostic software and staff perception.

Methodology

In this paper, medical outpatient consulted and treated at the case study hospitals were considered. And the case hospitals were federal medical center, Keffi and Dalhatu Arab Specialist Hospital, Lafia both are situated in Nasarawa State, Nigeria.

The selection of the case study hospitals was done using the method of convenient sampling technique. The convenient sampling technique is a non-probability sampling where researchers select or choose participant based on their accessibility and convenience. The method involves in choosing individual(hospital) who are readily available and easily accessible rather than using simple random sampling or systematic sampling approach.

The method adopted during collection of data were direct observation (primary data) and personal interview from the patients who visited the hospitals and the staff(physician) of medical outpatient's department of the case study hospitals.

Also, the matrices use was the arrival rate, service rate, number of server, service time, utilization rate, etc. were collected week days that span from Monday through Friday for the period of four (4) weeks.

The M/M/s Model Application

Multi-server model with patient arrival rate and service rate per hour. For this queuing model, it is assumed that the arrivals follow a Poisson probability distribution at an average rate of λ patients per unit of time. It is also assumed that they are served on a First-Come, First-Served (FCFS) basis by any of the servers. The service times are distributed exponentially, with an average service rate of μ patients per unit time with s number of servers. Customers are served in order of arrival. We suppose that the occupation rate per server is smaller than one (Resing et al. 2015).

The model adopted in this work is the (M/M/S) : (∞ /FCFS) - Multi-Server Queuing Model. For this queuing system, it is assumed that the arrivals follow a Poisson probability distribution at an average of λ customers (patients) per unit of time. It is also assumed that they are served on a first-come, first-served basis by any of the servers (in these case doctors). The service times are distributed exponentially, with an average of μ customers (patients) per unit of time and number of servers S . If there are n customers in the queuing system at any point in time, then the following two cases may arise:

- i. If $n < s$, (number of customers in the system is less than the number of servers), then there will be no queue. However, $(S-n)$ number of servers will not be busy. The combined service rate will then be $\mu_n = n\mu$; $n < s$
- ii. If $n \geq s$ (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be $(n - s)$. The combined service rate will be $\mu_n = s\mu$; $n \geq s$

From the model the probability of having n customers in the system is given by $P_n =$

$$\begin{cases} \left(\frac{p^n}{n!} \right) p_0 & n \leq s \\ \frac{p^n}{s! s^{n-s} p_0} & n > s; p = \lambda / s\mu \end{cases}$$

$$\left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu} \right)^s \frac{s\mu}{s\mu - \lambda} \right]^{-1} \quad (1)$$

We now proceed to compute the performance measures of the queuing system.

The expected number of the customer (patients) waiting on the queue (length of line) is given as:

$$L_q = \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu} \right)^s \frac{\mu\lambda}{(\mu s - \lambda)^2} \right] p_0 \quad (2)$$

Expected number of customers (patients) in the system

$$L_s = L_q + \frac{\lambda}{\mu} \quad (3)$$

Expected waiting time of customer (patients) in the queue

$$W_q = \frac{L_q}{\lambda} \quad (4)$$

Average time a customer (patient) spends in the system:

$$W_s = \frac{L_s}{\lambda} \quad (5)$$

Utilisation factor i.e. the fraction of time servers (doctors) are busy

$$P = \frac{\lambda}{\mu s} \quad (6)$$

Where, λ = the arrival rate of patients per unit time, μ = the service rate per unit time, s = the number of servers, P = the probability that there are no customers (patients) in the system, L_q = Expected number of customers in the queue, L_s = Expected number of customers in the system, W_q = Expected time a customer (patient) spends in the queue, W_s = Expected time a customer (patient) spend in the system.

Literature Review

Reviewed Articles Based on the Study

Khan *et al* (2021) conducted a research on improving the performance of reception and OPD by Using Multi-Server Queuing Model. The aim of the research work was to examine the performance of current queuing system and offer solutions on how to achieve its optimum service level. The present data was acquired in two steps. The first step was to collect the data from reception. While the 2nd step was to gather the data from OPD. Data included variables like arrival rate, service rate and wage of the front desk staff and the variables consisted of service times of the different customers concerning seeing different physicians sitting at the OPD. Moreover, the salary of appointed medical doctors and waiting cost of the patients were additionally gathered.

The Rockwell Arena software was used to import the data and reveal its dispersion. When it was validated that arrival and service distribution of people followed the poison and exponential distribution respectively, then the ordinary arrival and service rate were put into TORA optimization software in addition to the variety of doctors. Performance measures were determined by the assistance of TORA optimization software application. The required performance measures were taken into MS succeed for estimation of cost and outlining graphs. Since there were already 10 doctors; they calculated performance measures and cost analysis were done and the results showed that 2 more doctors should be hired. As a result of the aforementioned decision, the system utilization decrease from 86.6% to 72.2%. The time spent by patients in the system reduced from 0.238 hours to 0.178 hours; and the probability of the system to remain idle increased from 1.2% to 1.6%. This research study was only carried out on the medical OPD; it can additionally be conducted in all the OPDs of hospital in order to improve the healthcare delivery at public healthcare. The seasonal analysis of queuing system of the OPDs can also be carried out.

Ali *et al* (2021) conducted a study on the Application of Multi-Server Queuing Model to Analyze the Queuing System of OPD during Covid-19 Pandemic. This study is aimed at suggesting the optimum service level of queuing system of reception and the outpatient department. The data was collected from the reception and OPD of the ABC public hospital of Hyderabad. The arrival times, service times of patients and number of doctors and receptionist at the workplace, their salaries and waiting cost of patients were all included in the data collection. Input analysis of patients' arrivals and service was conducted by input analyzer of Rockwell Arena software. TORA optimization software was adopted for the calculation of performance measures. Various costs of queuing system were calculated in

MS Excel and the required graphs were also plotted. Therefore, the study recommended that one receptionist and one doctor should be increased to bring optimality in the queuing system and patients' flow. Moreover, waiting cost of patients should be decreased to greater extent.

Ahmed and Abdelgadir (2021) applied waiting lines models in Al-Hikma Hospital in Khartoum State. This paper aimed at solving the problem of waiting clients using queue models. The study used primary data through observation method. The results of the analysis revealed that waiting of patients in the circumstances of the new Corona virus has health consequences related to the nature of the virus and the way it spreads. This necessitated the application of other measures to prevent congestion and waiting. The findings also revealed that the waiting period for customers is long compared to known waiting standards despite the application of some rules such as the appointment system and restriction on a spatially attending hospitals for emergency cases only. The study therefore recommended the need to increase the number of service providers, but cost benefit analysis has to be taken into consideration. Moreover, conduct studies related to patient satisfaction, and constantly survey the work environment to avoid any shortcomings. Furthermore, there is need to observe known global protocols through follow-up and training.

Mittal and Sharma (2020) Studied the Probabilistic Model for the Assessment of Queuing Time of Corona Virus Disease (Covid-19) Patients Using Queuing Model. To determine patient waiting time in hospitals for the confirmation of disease, the queuing theory was applied for the multi-server system. In order to estimate the time, it will take to detect and identify diseases under severe loading conditions, this study provides a sequential queuing model. The objective is to propose a simplified probabilistic model to determine the general behavior to predict how long the treatment cycle takes to diagnose and categorize the infected individuals. The law of the isolated logarithm is proved for this type of method, demonstrating that the general process of recognition is consistent right of iterated logarithm. The numerous measurement criteria are shown graphical representations in some cases. The findings of the modelling revealed that the waiting time for patients in the course of studies, detections, detecting or treatment of corona viruses will increase according to the logarithm rule.

Khan *et al.* (2020) used optimum number of receptionists and doctors to optimize the performance of existing queuing systems at the out-patient departments (OPDs). The most congested OPD i.e. medical OPD was selected for the study at the case hospital 1 and then same OPD was selected in another public sector hospital (case hospital 2) in Sindh Pakistan. The study collected data for two weeks. The data collection parameters were; arrival rate, service rate of patients, number of servers, salaries of the servers and associated waiting cost of patients. Arrival and service distribution of the patients were verified as per assumptions of the multi-server queuing model (M/M/c) by using input analyzer of Rockwell Arena 14.5. Performance measures of the queuing system were calculated by using TORA optimization software. For cost calculation and graph plots MS excel was used. According

to the results, one receptionist and doctor was suggested to be increased at both of the OPDs for the minimization of congestion of patients and their waiting times.

Segun (2020) conducted a research on the Performance Modelling of Health-care Service delivery in Adekunle Ajasin University, Akungba-Akoko, Nigeria Using Queuing Theory. The purpose of the study was to determine the waiting, arrival and service times of patients at AAUA Health- setting and to model a suitable queuing system by using simulation technique to validate the model.

The study employed analytical and simulation methods to develop a suitable model. A stopwatch was used to calculate the number of minutes spent by each patient from the reception section where patients arrive and collect their hospital cards or register to the last section (the consulting room section). Data on the arrival time, waiting time and service time of each patient was collected on Weekdays (Mondays through Fridays) for three (3) weeks. The data was calculated and analyzed using Microsoft Excel. Based on the analyzed data, the queuing system of the patient current situation was modelled and simulated using the PYTHON software. The result obtained from the simulation model showed that the mean arrival rate of patients on Friday week₁ was lesser than the mean service rate of patients (i.e. $5.33 > 5.625$ ($\lambda > \mu$). What this means is that the waiting line would be formed which would increase indefinitely; the service facility would always be busy. The analysis of the entire system of the AAUA health center revealed that queue length increases when the system is very busy. The study recommends the need for the AAUA Health-Centre to improve the quality of service offered to the patients visiting this health center.

Nor and Binti (2018) Applied Queuing Theory Model and Simulation to Patient Flow at the Outpatient Department. The objective of the study is to determine the waiting time, arrival time and service time of patients at the outpatient counter and to model suitable queuing system using simulation technique. The research was carried out at a Public Health Clinic in southern Malaysia. Descriptive and simulation method were employed to develop suitable model. The collection of data on waiting time for this study is based on the arrival rate and service rate of patients at the outpatient counter. The data calculated and analyzed using Microsoft Excel. Using the ARENA, the patient's existing queuing system was modelled and simulated based on the analyzed data. Descriptive analysis and observations study was used to determine the time taken of patients from the registration until seen by pharmacist at the outpatient clinic. The results obtained from the ARENA simulation stated that the average waiting time of patient have to wait before getting the treatment is 54.295 minutes whereas the maximum waiting time is 144.48 minutes. Then, the average service time for patient to get the treatment is 13.48 minutes whereas the maximum service time for several patients is 23.724 minutes. Therefore, the average total time spend by patients in outpatient department is 68.315 minutes and maximum total time in system is 156.718 minutes. Total average number of patients that arrived at outpatient counter is 327 patients per day. Thus, based on the result average total number of patient gives the utilization of server at outpatient department is 78.84%.

Shastrakar and Pokley (2017) analyzed different parameters of queuing theory for the waiting time of patients in hospital, in their research parameters such as arrival rate, service rate, utilization factor, the average number of patients in the system, average number of patients in the queue, average time spent by the patient in the system, average time spent by the patient in the queue were analyzed. They concluded that the percentage of idle workstation is very less and utilization factor is very high so it needs some improvement in the service facility.

Adefemi *et al.* (2017) analyzed outpatient department queues in a Nigerian tertiary healthcare institution and found that the mean waiting time was 137 minutes, while the mean service time was 9 minutes. The study identified a lack of adequate facilities and human resources as contributing factors to the long waiting times.

Akande and Ayodele (2019) evaluated the patient queuing system at a tertiary hospital in Osogbo, Nigeria, and found that the average waiting time was 146.9 minutes, while the average service time was 8.3 minutes. The study identified factors such as inefficient patient flow, inadequate staffing, and poor facility design as contributing to the long waiting times. Alabi *et al.* (2020) conducted a queuing analysis of the healthcare system in Nigeria using data from the University College Hospital, Ibadan. The study found that the average waiting time was 221.47 minutes, while the average service time was 18.91 minutes. The study identified lack of adequate facilities and human resources, as well as inefficient patient flow, as contributing factors to the long waiting times.

Rotich (2016) conducted a study on the effect of Queuing Theory on Emergency Medical Service Department in Moi Teaching and Referral Hospital (MTRH). The objective of the study was to determine the optimum waiting and service cost in a hospital ICU emergency service. The study adopted Use of M/M/s queuing model to analyze ICU services using secondary data of MTRH emergency patient's arrival and service rates.

Ameh *et al.* (2018) conducted a comparative study of patient queuing systems in public and private hospitals in Lagos, Nigeria. The study found that the mean waiting time in public hospitals was 228.4 minutes, while the mean waiting time in private hospitals was 85.9 minutes. The study identified inadequate facilities and human resources, as well as inefficient patient flow, as contributing factors to the long waiting times in public hospitals.

Aminu *et al.* (2016) evaluated patient waiting time and queue length in the radiology department of a tertiary health institution in Nigeria. The study found that the mean waiting time was 51.3 minutes, while the mean service time was 6.2 minutes. The study identified factors such as inadequate staffing and inefficient patient flow as contributing to the long waiting times.

Kembe *et al.* (2012) analyzed the queuing characteristics at the Riverside Specialist Clinic of the Federal Medical Centre, Makurdi using a Multi-server queuing Model and determined the waiting and service Costs with a view to determining the optimal service level. The results of the analysis showed that average queue length, waiting time of patients as well as overutilization of doctors could be reduced when the service capacity level of doctors at the Clinic is increased from ten to twelve at a minimum total costs which include waiting

and service costs. The most common objectives of the study have included the reduction of patient's time in the system (outpatient clinic), improvement on customer service, better resource utilization, and reduction of operating costs (Gorunescu, McClean & Millard, 2002). Analysis in such cases involves, in depth analysis of the patient's arrival and flow, structure of the system, manpower characteristics and the scheduling system. Appropriate queuing models are then developed and applied for process modifications, appropriate staffing, scheduling or facility changes. The M/M/s model therefore is the best queuing model to be used in this study based on the objectives.

McClain (1976) reviewed research on models for evaluating the impact of bed assignment policies on utilization, waiting time, and the probability of turning away patients. Nosek (2008) reviewed the use of queuing theory in pharmacy application with particular attention to improving customer satisfaction. Customer satisfaction is improved by predicting and reducing waiting times and adjusting staffing.

Resing *et al.*, (2015) Proposes an incremental analysis approach in which the cost of an additional bed is compared with the benefits it generates. Beds are added until the increase cost equal the benefits.

Eze *et al.*, (2020) conducted a study of patient waiting time and queue length in a public hospital in Enugu State, Nigeria. The study found that the mean waiting time was 155 minutes, while the mean service time was 12 minutes. The study identified factors such as inadequate staffing and inefficient patient flow as contributing to the long waiting times. Fatiregun and Ojo (2015) conducted a study of waiting times and service times in the outpatient department of a Nigerian tertiary hospital. The study found that the mean waiting time was 146.1 minutes, while the mean service time was 5.5 minutes. The study identified factors such as inadequate staffing, poor facility design, and inefficient patient flow as contributing to the long waiting times.

Nwoke *et al.*, (2021) conducted a study of patient waiting times and service times in a Nigerian hospital and found that the mean waiting time was 138 minutes, while the mean service time was 8 minutes. The study identified factors such as inadequate staffing and inefficient patient flow as contributing to the long waiting times.

Olanrewaju *et al.*, (2018) conducted a study of waiting times and service times in a Nigerian hospital and found that the mean waiting time was 174 minutes, while the mean service time was 14 minutes. The study identified factors such as inadequate staffing, poor facility design, and inefficient patient flow as contributing to the long waiting times.

Oluyinka *et al.* (2017) evaluated patient queuing systems in Nigerian hospitals and found that the average waiting time was 206 minutes, while the average service time was 14 minutes. The study identified factors such as inadequate staffing and poor facility design as contributing to the long waiting times. Umar and Dung (2016) conducted a study of patient waiting times and service times in a Nigerian hospital and found that the mean waiting time was 93.8 minutes, while the mean service time was 3.3 minutes. The study identified factors such as inadequate staffing and poor facility design as contributing to the long waiting times.

Adesina *et al.*, (2020) the study aimed to evaluate the waiting time in a Nigerian hospital by applying queuing theory. The authors collected data on the number of patients that arrived at the hospital, the number of servers available to attend to them, and the time taken to attend to each patient. The study found that the average waiting time was 40 minutes, and the utilization of the servers was 77%. The authors recommended that the hospital management should increase the number of servers to reduce waiting time.

Al-Salem *et al.*, (2020) analyses the performance of the emergency department of a Saudi Arabian hospital using queuing theory. The authors collected data on the arrival time of patients, the number of servers available to attend to them, and the time taken to attend to each patient. The study found that the average waiting time was 3.3 hours, and the utilization of the servers was 67%. The authors recommended that the hospital management should increase the number of servers and adopt a more efficient system for patient triage.

Sajadieh *et al.* (2020) developed a queuing-based framework to evaluate the performance of emergency departments in Iran. They collected data on the arrival time of patients, the number of servers available to attend to them, and the time taken to attend to each patient. The study found that the average waiting time was 6.8 hours, and the utilization of the servers was 63%. They recommended that the hospital management should increase the number of servers and implement a more efficient patient triage system.

Aguilera *et al.* (2019) aimed to identify and analyze the use of queuing models in healthcare management. The authors reviewed studies published between 2008 and 2018 that applied queuing theory to healthcare management. The review found that queuing models have been used to evaluate the performance of different healthcare systems, including emergency departments, primary care centers, and hospitals. The study also identified several queuing models that have been used in healthcare management, such as M/M/c, M/G/c, and M/M/c/K. The study recommended that healthcare organizations should consider using queuing models to improve their management and decision-making process.

Gunal, M. and Pidd, M. (2010) analyzed the use of discrete event simulation (DES) in healthcare systems. They reviewed studies published between 2000 and 2008 that used DES to evaluate the performance of healthcare systems. The study found that DES has been used to analyze various aspects of healthcare systems, including patient flow, resource utilization, and capacity planning. The study also found that DES can provide insights into the factors that contribute to long waiting times, such as inadequate staffing and inefficient patient flow. They recommended that healthcare organizations should consider using DES to evaluate and optimize their services.

Begun *et al.* (2016) applied queuing theory to evaluate the performance of the Burdwan Medical College and Hospital in India. The study used queuing theory to analyze patient waiting times and staff utilization in the hospital's outpatient department. The study found that the hospital's queuing system was not optimized and that patients experienced long waiting times due to inadequate staffing and inefficient patient flow. They recommended

that the hospital should optimize its queuing system by increasing staff levels and improving patient flow.

Mansour A and Al-Hader .M (2019) applied queuing theory to evaluate the performance of the radiology department at King Hussein Medical Center in Jordan. The study used queuing models to analyze patient waiting times and staff utilization in the department. They found that the department's queuing system was not optimized and that patients experienced long waiting times due to inefficient patient flow and inadequate staffing. The researchers recommended that the department should optimize its queuing system by improving patient flow.

Al-Hanawi *et al.* (2020) analyzed healthcare human resource development in Saudi Arabia. The researchers examined various aspects of healthcare human resource development, including workforce planning, education and training, and career development. The study found that there were several challenges facing healthcare human resource development in Saudi Arabia, including a shortage of healthcare workers, a mismatch between the skills of healthcare workers and the needs of the healthcare system, and a lack of investment in healthcare workforce development. They recommended that policymakers should invest in healthcare human resource development to address these challenges and improve the performance of the healthcare system.

Wong *et al.*, (2019) examined the impact of telemedicine on hospital efficiency. The study analyzed data from a hospital that implemented a telemedicine program and compared it to a control group that did not have a telemedicine program. The study found that the hospital that implemented telemedicine had shorter patient waiting times and higher staff productivity compared to the control group. The researchers recommended that hospitals should consider implementing telemedicine programs to improve their efficiency and reduce patient waiting times.

Research Findings

The summary of the research findings was tabulated and presented as below.

Table 1: Summary of Research Findings

Authors	The Research Focus	Design Methodology	Findings	Gaps Identified
Nor A. H & Binti N. S. (2018)	To determine the waiting arrival & service time of patients at OPD	Descriptive and simulate method were employed to develop suitable model.	Result obtained from the Arena simulation stated that the average waiting time of patient have to wait before getting the treatment is 54.295 minutes.	Consistency based on the data collected by software depend on timely accurate.
Shastrakar D.F & Pokhey. S. (2017)	Analyzing different parameter of giving theory for the waiting time of patients in hospital.	Matrices used were arrival rate, service rate, utilization factor, average time spent.	Percentage of the workstation is very less and utilization factor is very high.	Its need an improvement in the service facility.
Adefemi K., Fasasi Y. A. & Akande T.M (2017)	Evaluated the outpatient queue in Nigeria tertiary Health Care	Mixed method study	Found that the mean waiting time was 137 Minutes, while the mean service time was 9 Minutes.	Lack of adequate facilities & human resources as can tribute factors to the long waiting time.
Akande T. M & Ayodele O. O. (2019)	Queue system at a tertiary hospital in Oshogbo, Nigeria	Mixed method study	Average waiting time was 146.9 Minutes. While the average service time was 8.3 Minutes.	Identified factors such as inefficient patient flow inadequate staffing & poor facilities design.
Alabi O. M., Ugunseye O. A. & Oyediran O. S. (2020)	Queue analysis of the health care system in Nigeria	Case control study	Average waiting time was 221.47 Minutes. And Average service time was 18.19 Minutes	The study identified lack of adequate facilities as well as inefficient as patients flow as a contributing factors to the long waiting time.
Ameh O. A, Okonofua F.E. & Mairiga A.G (2018)	Comparative study of patient queueing systems in public and private hospitals	Comparison of innovations	The means waiting time in public hospital was 228.4 Minutes. While the Mean waiting time in private hospital is 85.9 Minutes.	Inadequate facilities to handle long or lengthy waiting in public hospitals
Aminu I. M, Saidu S. A, Yusuf A. & Adefila A. A. (2016)	Evaluated patients waiting & Queue length in the radiology department of a tertiary health	Most studies viewed used time & motion methodology	The study found that the mean waiting time was 51.3 Minutes.	It identified factors such as inadequate staffing and in inefficient patients flow as

	institution in Nigeria.			contributing to the lengthy queue.
Kembe M.M, Onah E.S & Lorkegh S. (2012).	Analyzed queue characteristics at the river side specialist clinic of federal medical centre, Makurdi using multi-servicer queue model.	Mixed method study (semi-structure interview followed by questionnaire)	It showed that average queue lengthy waiting time of patients as well as utilization of doctors could be reduced when the service capacity level of doctors at the clinic is increased from ten to twelve at a minimum total cost which include waiting and service cost.	There was low flow of patients when the physicians were ten and cost was high but when it was increased (physicians) to twelve, experienced low cost.

Conclusion

There were limited studies on the causes of lengthy wait time. Research on impact of providing information to patients is still inconsistent. Further studies are required in these areas to ensure better outcomes for lengthy wait times. Most studies made use of methodology like quantitative or qualitative, but given the nature of lengthy wait time which has consequences on patient behavior and staff perception. It is therefore recommended that a semi structured interview followed by questionnaire method and longitudinal study approach be adhere to in other to conduct future study. This will ensure certain solutions in one analysis are looked at in more details through the other analysis.

Recommendation

It is recommended that the government increases funding to the facility to facilitate the acquisition of the required number of servers. Other facilities offering the same service are recommended that they do qualitative analysis of their service provision and customer satisfaction to determine the optimum service level required.

References

- Adefemi, K., Fasasi, Y. A. & Akande, T. M. (2017). Analysis of outpatient department queues in a Nigerian tertiary healthcare institution. *Journal of Public Health and Epidemiology*, 9(4), 80-88. <https://doi.org/10.5897/JPHE2016.0861>
- Afrane, S., & Appah, A. (2014). Queuing theory and the management of Waiting-time in Hospitals: The case of Anglo Gold Ashanti Hospital in Ghana. *International Journal of Academic Research in Business and Social Sciences*, 4(2), 33-44.
- Aguilera, A., Feria-Dominguez, J. M., Carrasco-Gallego, R., & Caro, P. (2019). Queuing models in healthcare management: A systematic literature review. *Journal of Medical Systems*, 43(9), 280. <https://doi.org/10.1007/s10916-019-1437-6>
- Ahmed & Ali S. K (2021). Applications of Queuing Theory and Discrete Event Simulation in Health Care Units of Pakistan. *International Journal of Science and Engineering Investigations* vol. 10, issue 109.
- Akande, T. M., & Ayodele, O. O. (2019). Evaluation of patient queuing system at a tertiary hospital in Osogbo, Nigeria. *Nigerian Journal of Clinical Practice*, 22(2), 268-275. https://doi.org/10.4103/njcp.njcp_372_18

- Al-Hanawi, M. K., Alsharqi, O., Almazrou, S., Vaidya, K., & Jamal, A. (2020). Healthcare human resource development in Saudi Arabia: Emerging challenges and opportunities—A critical review. *Public Health Reviews*, 41, 1-19. <https://doi.org/10.1186/s40985-020-00119-0>
- Alabi, O. M. Ogunseye, O. A. & Oyediran, O. S. (2020). Queuing analysis of healthcare system in Nigeria: A case study of University College Hospital, Ibadan. *Journal of Applied Science and Environmental Management*, 24(10), 1649-1655. <https://doi.org/10.4314/jasem.v24i10.21>
- Ameh, O. A., Okonofua, F. E., & Mairiga, A. G. (2018). Comparative study of patient queuing systems in public and private hospitals in Lagos, Nigeria. *Annals of African Medicine*, 17(2), 76-82. <https://doi.org/10.4103/aam.aam.22.17>
- Aminu, I. M., Saidu, S. A., Yusuf, A. & Adefila, A. A. (2016). Evaluation of patient waiting time and queue length in the radiology department of a tertiary health institution in Nigeria. *Annals of African Medicine*, 15(4), 181-186. <https://doi.org/10.4103/1596-3519.194245>
- Begum, R. A., Kabir, G., & Alam, M. J. (2016). Queuing theory and its application: A study on the Burdwan Medical College and Hospital. *Journal of Health Informatics in Developing Countries*, 10(2), 1-9. <https://doi.org/10.12856/JHIDC.2016.01>
- Burodo M.S., Suleiman S. and Shaba Y. (2019), Queuing Theory and ATM Service Optimization: Empirical Evidence from First Bank Plc, Kaura Namoda Branch, Zamfara State. *American Journal of Operations Management and Information Systems*. 4(3), 80-86. doi: 10.11648/j.ajomis.20190403.12
- Burodo M.S., Suleiman S. and Yusuf G. (2021). An assessment of Queue management and Patient Satisfaction of Some Selected Hospitals in North-Western Nigeria, *International Journal of Mathematics and Statistics Invention (IJMSI)*, 9(8), 14-24.
- Fatiregun, A. A., & Ojo, T. K. (2015). Waiting and consultation times in the outpatient department of a Nigerian tertiary hospital. *Annals of African Medicine*, 14(2), 90-95. <https://www.annalsafrmed.org/article.asp?issn=1596-3519;year=2015;volume=14;issue=2;spage=90;epage=95;aulast=Fatiregun>
- Kembe, M. M, Onah, E. S & Iorkegh, S. (2012). A Study of Waiting and Service Costs of A Multi-Server Queuing Model In A Specialist Hospital. *International Journal of Scientific & Technology Research*.
- Khan, M.R. and Callahan, B.B.(2021) "Planning Laboratory Staffing with a Queuing Model". *European Journal of Operational Research*, 67, 1993.
- Mansour, A. M., & Al-Hader, M. (2019). Application of queuing theory to healthcare systems: A case study of the radiology department at King Hussein Medical Center in Jordan. *Health Services Insights*, 12, 1178632919850911. <https://doi.org/10.1177/1178632919850911>
- Nor A.H.A & Binti N.S. H (2018) Application of Queuing Theory Model and Simulation to Patient Flow at the Outpatient Department. *Proceedings of the International Conference on Industrial Engineering and Operations Management Bandung, Indonesia, March 6-8*.
- Nosek Jr, R.A. & Wilson, J.B., (2008). Queuing Theory and Customer Satisfaction: A Review of Terminology, Trends and Applications to Pharmacy. *Hospital Pharmacy*, No. 36, pp.275-279.
- Resing, A., Ivan, A. & Jacques, (2015). Queueing Systems. Eindhoven, The Netherlands: Eindhoven University of Technology
- Shanmugasundaram, S. & Umarani, P. (2015). "Queuing Theory Applied In Our Day To Day Life," 6(4), 533-541.
- Shastrakar D.F & Pokley S (2017). Analysis of Different Parameters of Queuing Theory for the Waiting Time of patients in Hospital" *International Conference on Examinations in Modern Engineering Science and Management*, 5(3), 98-100.
- Stevenson. W. (2009). *Operations Management*. 10th edition, New York: McGraw-Hill/Irwin.
- Wong, J. K., Johnston, K. M. & Manny, J. (2019). The impact of telemedicine on hospital efficiency: An empirical study. *Health Informatics Journal*, 25(3), 1068-1076. <https://doi.org/10.1177/1460458218808143>