

Application of Pair Fully Fuzzy Matrix Equation in Medical Diagnosis under Fuzzy Environments

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Abstract

In this paper, patients with current states and recovery state have been solved simultaneously to show the rate at which the malaria fever and viral fever reduced in the patients. The patients with current states and recovery state in the field of medical diagnosis can be made using fuzzy kronecker product, vec operator, associated linear system and pseudo inverse. Examples are illustrated to verify the proposed approach and the obtained results are discussed.

Keywords: Medical diagnosis, Fully fuzzy matrix Equation, Triangular fuzzy number, Associated linear system.

Introduction

The concept of fuzzy set and fuzzy number was introduced by Zadeh [17]. Recently fuzzy set theory is based on developing knowledge-based system in medicine for the tasks of medical findings. In real life situations, the imprecise nature of medical documentation and uncertain information gathered for decision making requires the use of "fuzzy". The physicians generally gather knowledge about the patient from the past history, laboratory test result and other investigative procedures such as x-rays and ultra sonic rays etc. The knowledge provided by each source carries with it varying degrees of uncertainty. Thus, the best and most useful descriptions of diseases entities often use linguistic terms that are vague.

There are variety of models involving fully fuzzy matrices to deal with different complicated aspects of medical diagnosis. (Sanchez, 1976) formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between symptoms and diseases. (Meenakshi and Kaliraja, 2011), have extended sanchez's approach for medical diagnosis using the representation of a interval valued fuzzy matrix. They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of medical diagnosis on it. Also, (T. Geetha and A. Usha, 2020) considered fuzzy matrix for medical application. They used intuitionist fuzzy matrix of triangular Atanassov. This study aims to use the algorithm of (Daud et al., 2018) to find the solution of the data from the medical problem of the fully fuzzy matrix equation below.

$$\tilde{A}_m \tilde{X}_m = \tilde{B}_m \quad (1)$$

In section 2, the fundamental concept of fuzzy set theory and kronecker operation are provided. In section 3, an algorithm is presented for medical diagnosis which is an extension (Daud et al., 2019) approach with modified procedure using fuzzy kronecker product and fuzzy vec-operator is utilized to transform the *fully fuzzy matrix equation* into simpler form of equations then a pseudo inverse method is used to get the actual solution. In section 4, a numerical example is illustrated to demonstrate the proposed algorithm. Finally, the conclusion is drawn in section 5.

Preliminaries

This section will recall some definitions and theorems that will be used in this study.

Definition 2.1. (Dubois and Prade, 1978). The arithmetic operation of two fuzzy numbers $\tilde{M} = (m, \alpha, \beta)$ and $\tilde{N} = (n, \gamma, \delta)$ are as follows:

1. Sum

$$\tilde{M} \oplus \tilde{N} = (m, \alpha, \beta) \oplus (n, \gamma, \delta) = (m + n, \alpha + \gamma, \beta + \delta) \quad (2)$$

2. Opposite:

$$-\tilde{M} = -(m, \alpha, \beta) = (-m, \beta, \alpha) \quad (3)$$

3. Difference

$$\tilde{M} \ominus \tilde{N} = (m, \alpha, \beta) \ominus (n, \gamma, \delta) = (m - n, \alpha + \delta, \beta + \gamma) \quad (4)$$

4. Product:

$$\tilde{M} \otimes \tilde{N} = (m, \alpha, \beta) \otimes (n, \gamma, \delta) = (mn, m\gamma + n\alpha, m\delta + n\beta) \quad (5)$$

Definition 2.2: (Dehghan et al., 2006) A positive fuzzy number $\tilde{X} = (x, y, z)$, is solution of fully fuzzy linear system, $\tilde{A}\tilde{X} = \tilde{B}$, where $\tilde{A} = (A, M, N) \geq 0$ and $\tilde{B} = (b, h, g) \geq 0$, i.e.

$$(A, M, N) \otimes (x, y, z) = (b, h, g)$$

If

$$\begin{cases} Ax = b \\ Ay + Mx = h \\ Az + Nx = g \end{cases} \quad (6)$$

Definition 2.3 (Malkawi et al., 2015):- Let $[\tilde{A}] = (\tilde{a}_{ij})_{m \times n}$ be a fuzzy matrix. The transpose fuzzy matrix $\tilde{A}^T$ is denoted as follows

$$\tilde{A}^T = (\tilde{a}_{ji})_{n \times m} \quad (7)$$

Definition 2.4 (Malkawi et al., 2014): A fully fuzzy matrix equation is a matrix equation whereby the matrix and the element of the vector are fully fuzzy.

Definition 2.5 (Malkawi et al., 2015): Fuzzy kronecker product of $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$ and $\tilde{B} = (\tilde{b}_{ij})_{p \times q}$ is represent as

$$\tilde{A} \otimes_k \tilde{B} = [\tilde{a}_{ij}\tilde{B}]_{(mp) \times (nq)}$$

$$\tilde{A} \otimes_k \tilde{B} = \begin{bmatrix} \tilde{a}_{11}\tilde{B} & \tilde{a}_{12}\tilde{B} & \dots & \tilde{a}_{1n}\tilde{B} \\ \tilde{a}_{12}\tilde{B} & \tilde{a}_{12}\tilde{B} & \dots & \tilde{a}_{12}\tilde{B} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{12}\tilde{B} & \tilde{a}_{12}\tilde{B} & \dots & \tilde{a}_{12}\tilde{B} \end{bmatrix} \quad (8)$$

Definition 2.6 (Malkawi et al., 2015): Vec-operator of a fuzzy matrix is a linear transformation that converts the fuzzy matrix $\tilde{C} = (\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_s)$ into a column vector

$$Vec(\tilde{C}) = \begin{bmatrix} \tilde{c}_1 \\ \tilde{c}_2 \\ \vdots \\ \tilde{c}_n \end{bmatrix} \quad (9)$$

Definition 2.7: (Herdelberz, 2005) The unitary fuzzy matrix is a square matrix defined as $\tilde{U}_n = (\tilde{U}_{i,j})_{n \times n}$

$$\tilde{U}_{ji} = \begin{cases} (0,0,0) & i \neq j \\ (1,0,0) & i = j \end{cases}$$

In matrix form, $\tilde{U}_n = (\tilde{U}_{i,j})_{n \times n}$ is represented as follows

$$\tilde{U} = \begin{bmatrix} (1,0,0) & (0,0,0) & \dots & (0,0,0) \\ (0,0,0) & (1,0,0) & \dots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \dots & (1,0,0) \end{bmatrix} \quad (10)$$

Definition 2.8 (Moore et al., 1920): Pseudoinverse is a matrix inverse- like object defined for a complex matrix even if it is not necessarily square.

Theorem 1: Malkawi et al. (2015). If $\tilde{A} = (A_{ij})_{m \times m}$ is a fuzzy matrix and $\tilde{U} = (\tilde{U}_{ij})_{p \times p}$ is a unitary fuzzy matrix, then,

- i. $\tilde{A}\tilde{U} = \tilde{U}\tilde{A} = \tilde{A}$
 - ii. $\tilde{U}^T = \tilde{U}$
- (11)

Proof:- i. $\tilde{A}\tilde{U} = \tilde{A}$

$$\begin{aligned} \tilde{A}\tilde{U} &= \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1m} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \tilde{a}_{nm} \end{bmatrix} \begin{bmatrix} (1,0,0) & (0,0,0) & \dots & (0,0,0) \\ (0,0,0) & (1,0,0) & \dots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \dots & (1,0,0) \end{bmatrix} \\ &= \begin{bmatrix} \tilde{a}_{11} + 0 + \dots + 0 & 0 + \tilde{a}_{12} + \dots + 0 & \dots & 0 + 0 + \dots + \tilde{a}_{1m} \\ \tilde{a}_{21} + 0 + \dots + 0 & 0 + \tilde{a}_{22} + \dots + 0 & \dots & 0 + 0 + \dots + \tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} + 0 + \dots + 0 & 0 + \tilde{a}_{n2} + \dots + 0 & \dots & 0 + 0 + \dots + \tilde{a}_{nm} \end{bmatrix} \\ &= \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1m} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \tilde{a}_{nm} \end{bmatrix} = \tilde{A} \end{aligned}$$

Hence, $\tilde{A}\tilde{U} = \tilde{A}$

Similarly, $\tilde{U}\tilde{A} = \tilde{A}$.

ii. $\tilde{U} = \tilde{U}^T$

$$\tilde{U}_{ij} = \begin{bmatrix} (1,0,0) & (0,0,0) & \dots & (0,0,0) \\ (0,0,0) & (1,0,0) & \dots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \dots & (1,0,0) \end{bmatrix}, \tilde{U}_{ji} = \begin{bmatrix} (1,0,0) & (0,0,0) & \dots & (0,0,0) \\ (0,0,0) & (1,0,0) & \dots & (0,0,0) \\ \vdots & \vdots & \ddots & \vdots \\ (0,0,0) & (0,0,0) & \dots & (1,0,0) \end{bmatrix} = \tilde{U}^T,$$

$\forall i \neq j$. Q. E. D

Theorem 2: Malkawi *et al.* (2015). Let $\tilde{P} = (\tilde{p}_{ij})_{m \times m}$, $\tilde{X} = (\tilde{x}_{ij})_{m \times n}$ and $(\tilde{U}_{ij})_{n \times n}$, where $\tilde{a}_{ij}, \tilde{x}_{ij} \in F[\mathbb{R}]$, where $\tilde{U}$ is a unitary fuzzy matrix. Then

$$\tilde{P}\tilde{X} = \text{Vec}(\tilde{P}\tilde{X}) = (\tilde{U} \otimes_k \tilde{P})\text{Vec}(\tilde{X}) \quad (12)$$

Proof: Using theorem 1, $\tilde{P}\tilde{X} = \tilde{P}\tilde{X}\tilde{U}$ then $\text{Vec}(\tilde{P}\tilde{X}) = \text{Vec}(\tilde{P}\tilde{X}\tilde{U}) = (\tilde{U}_n \otimes_k \tilde{P})\text{Vec}(\tilde{X})$

However, since $\tilde{U}^T = \tilde{U}$, then $\text{Vec}(\tilde{P}\tilde{X}) = (\tilde{U}_n \otimes_k \tilde{P})\text{Vec}(\tilde{X})$ Q. E. D

Theorem 3: Malkawi *et al.* (2015). Let $\tilde{Q} = (\tilde{q}_{ij})_{n \times n}$, $\tilde{X} = (\tilde{x}_{ij})_{m \times n}$ and $(\tilde{U}_{ij})_{m \times m}$, where $\tilde{q}_{ij}, \tilde{x}_{ij} \in F[\mathbb{R}]$, where $\tilde{U}$ is a unitary fuzzy matrix. Then $\text{Vec}(\tilde{X}\tilde{Q}) = (\tilde{Q}^T \otimes_k \tilde{U}_m)\text{Vec}(\tilde{X})$ (13)

Proof: Since $\tilde{X}\tilde{Q} = \tilde{U}\tilde{X}\tilde{Q}$

$$\text{Vec}(\tilde{X}\tilde{Q}) = \text{Vec}(\tilde{U}\tilde{X}\tilde{Q}) = (\tilde{Q}^T \otimes_k \tilde{U}_m)\text{Vec}(\tilde{X}) \quad \text{Q. E. D}$$

Theorem 4: Malkawi *et al.* (2015). let $\tilde{P} = (\tilde{p}_{ij})_{m \times m}$, $\tilde{Q} = (\tilde{q}_{ij})_{n \times n}$ and $\tilde{X} = (\tilde{x}_{ij})_{m \times n}$. Then the fully fuzzy matrix equation $\tilde{P}\tilde{X} + \tilde{X}\tilde{Q} = \tilde{C}$ can be transformed into fully fuzzy linear equation:

$$[(\tilde{U}_n \otimes_k \tilde{P}) + (\tilde{Q}^T \otimes_k \tilde{U}_m)]\text{Vec}(\tilde{X}) = \text{Vec}(\tilde{C}) \quad (3.4)$$

where $\tilde{U}_m$ and $\tilde{U}_n$ denote the fuzzy unitary matrices order m and n, respectively.

Proof:- Using Eq. (3.1) $\tilde{P}\tilde{X} + \tilde{X}\tilde{Q} = \tilde{C}$ can be written as $\tilde{P}\tilde{X}\tilde{U} + \tilde{U}\tilde{X}\tilde{Q} = \tilde{C}$, then

$$\text{Vec}(\tilde{P}\tilde{X} + \tilde{X}\tilde{Q}) = \text{Vec}(\tilde{C})$$

$$\text{Vec}(\tilde{P}\tilde{X}\tilde{U} + \tilde{U}\tilde{X}\tilde{Q}) = \text{Vec}(\tilde{C})$$

$$\text{Vec}(\tilde{P}\tilde{X}\tilde{U}) + \text{Vec}(\tilde{U}\tilde{X}\tilde{Q}) = \text{Vec}(\tilde{C})$$

However, since $\tilde{U}^T = \tilde{U}$, then

$$(\tilde{U}_n^T \otimes_k \tilde{P})\text{Vec}(\tilde{X}) + (\tilde{Q}^T \otimes_k \tilde{U}_m)\text{Vec}(\tilde{X}) = \text{Vec}(\tilde{C})$$

$$[(\tilde{U}_n \otimes_k \tilde{P}) + (\tilde{Q}^T \otimes_k \tilde{U}_m)]\text{Vec}(\tilde{X}) = \text{Vec}(\tilde{C}) \quad \text{Q. E. D}$$

Theorem 5: Daud *et al.* (2019). let $\tilde{P} = (\tilde{a}_{ij})_{m \times m}$, $\tilde{Q} = (\tilde{q}_{ij})_{n \times n}$. Then

$$\tilde{P} \otimes_k \tilde{Q} = [(\tilde{P} \otimes \tilde{U})(\tilde{U} \otimes \tilde{Q})] \quad (15)$$

Theorem 6: Daud *et al.* (2019). let $\tilde{P} = (\tilde{p}_{ij})_{p \times p}$, $\tilde{Q} = (\tilde{q}_{ij})_{r \times s}$ and $\tilde{X} = (\tilde{x}_{ij})_{q \times r}$. Then the fully fuzzy matrix equation $\tilde{P}\tilde{X}\tilde{Q}$ can be transformed into fully fuzzy linear equation.

$$\text{Vec}(\tilde{P}\tilde{X}\tilde{Q}) = (\tilde{P}^T \otimes_k \tilde{Q})\text{Vec}(\tilde{X}) \quad (16)$$

Proof: use Eq. (1) to write a matrix $\tilde{X}$ as $\tilde{X} = \tilde{X}\tilde{U}$ and also from Eq. (1) is

$\text{Vec}(\tilde{P}\tilde{X}\tilde{Q}) = \text{Vec}(\tilde{P}\tilde{X}\tilde{U}^T\tilde{Q})$. By matrix transpose is $\text{Vec}(\tilde{P}\tilde{X})(\tilde{U}^T\tilde{Q}^T)$

$(\tilde{Q}^T\tilde{U} \otimes_k \tilde{U})\text{Vec}(\tilde{P}\tilde{X}) = (\tilde{Q}^T \otimes_k \tilde{U})\text{Vec}(\tilde{P}\tilde{X})$

From Eq. (2) $= (\tilde{Q}^T \otimes_k \tilde{U})[(\tilde{U} \otimes_k \tilde{P})\text{Vec}(\tilde{X})]$
 $= [(\tilde{Q}^T \otimes_k \tilde{U})(\tilde{U} \otimes_k \tilde{P})]\text{Vec}(\tilde{X})$

From Eq. (5) $= (\tilde{Q}^T \otimes_k \tilde{P})\text{Vec}(\tilde{X})$

Q. E. D

Therefore, the proposition is obviously proved.

Theorem 7: Malkawi *et al.* (2014) An associated linear system is a system of linear equation $AX = B$, which is associated linear system of the FFLS, $\tilde{A}\tilde{X}=\tilde{B}$, where

$$A = \begin{pmatrix} F & 0 & 0 \\ M & F & 0 \\ N & 0 & F \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B = \begin{pmatrix} b \\ h \\ g \end{pmatrix}, \quad (17)$$

where F, M and N are rectangular matrices with common size of n , whereas x, y, z, b, h and g are vectors of $m \times n$ components.

Theoretical Foundation

In this section, theoretical foundation is built which lead to the development of the algorithm in solving fully fuzzy matrix equation.

A triangular fuzzy matrix is used to modify the procedure. Considering the following:

Let S be the symptoms of certain diseases, D is set of diseases and P is set of patients. The elements of triangular fuzzy number matrix are defined as

$$A = (a_{i,j})_{m \times l} \text{ where } a_{i,j} = (a_{i,j}\alpha, a_{i,j}m, a_{i,j}\beta) \text{ is the } i, j^{th} \text{ element of } A, \\ 0 \leq a_{i,j}\alpha \leq a_{i,j}m, \leq a_{i,j}\beta \leq 1$$

Here, $a_{i,j}\alpha$, is the lower bound, $a_{i,j}m$, is the moderate value and $a_{i,j}\beta$ is the upper bound.

Suppose there are three patients P_1, P_2, P_3 and P_4 in a hospital with symptoms temperature, headache, cough and stomach problems. Let the possible diseases relate to the above symptoms be viral fever and malaria.

Now, we consider the set $S = \{s_1, s_2, s_3, s_4\}$ as universal set, where s_1, s_2, s_3 and s_4 represent the symptoms temperature, headache, cough and stomach problem respectively and the set $D = \{d_1, d_2\}$ where d_1 and d_2 represent the parameters viral fever and malaria respectively.

Suppose there are four patients P_1, P_2, P_3 and P_4 in a hospital with symptoms temperature, headache, cough and stomach problems. Let the possible diseases relate to the above symptoms be viral fever and malaria.

Let F_{mm} and F_{mn} denote the set of all $m \times m$ and $m \times n$ matrices respectively over F .

We consider the set $S = \{s_t, s_h, s_c, s_s\}$ as universal set, where s_t, s_h, s_c and s_s represent temperature, headache, cough and stomach problem respectively and the set $D = \{d_v, d_m\}$ where d_v and d_m represent the parameter viral fever and malaria respectively. Suppose that

$$F(d_v) = \begin{pmatrix} (0.75, 0.50, 0.80) \\ (0.25, 0.20, 0.30) \\ (0.55, 0.45, 0.60) \\ (0.30, 0.25, 0.40) \end{pmatrix}, \quad F(d_m) = \begin{pmatrix} (0.75, 0.60, 1.0) \\ (0.50, 0.35, 1.0) \\ (0.75, 0.70, 1.0) \\ (0.45, 0.75, 1.0) \end{pmatrix}$$

The triangular fuzzy number matrix (F, D) is a parameterized family $(F(d_v), (F(d_m)))$ of all triangular fuzzy number matrix over the set S and are determined from export medical documentation Elizebeth *et al.* (2013) and (Venketsan *et al.*, 2019). But the element of triangular fuzzy number matrix in Elizebeth *et al.* (2013) is $0 \leq a_{i,j}\alpha \leq a_{i,j}m, \leq a_{i,j}\beta \leq 10$. Thus, the triangular fuzzy number matrix (F, D) represent the relation matrix R_0 , where R_0 is called the symptoms - diseases matrix given by

$$R_0 = \begin{matrix} & d_v & d_m \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix} & \begin{pmatrix} (0.75, 0.50, 0.80) & (0.75, 0.60, 1.0) \\ (0.25, 0.20, 0.30) & (0.50, 0.35, 1.0) \\ (0.55, 0.45, 0.60) & (0.75, 0.70, 1.0) \\ (0.30, 0.25, 0.40) & (0.45, 0.75, 1.0) \end{pmatrix} \end{matrix}$$

Again we take $P = \{p_1, p_2, p_3, p_4\}$ as universal set, where p_1, p_2, p_3 and p_4 represent patients respectively and $S = \{s_t, s_h, s_c, s_s\}$ as a set of parameters. Suppose that,

$$G(s_t) = \begin{pmatrix} (0.35, 0.20, 0.45) \\ (0.40, 0.30, 0.45) \\ (0.25, 0.30, 0.4) \\ (0.20, 0.25, 0.50) \end{pmatrix}, \quad G(s_h) = \begin{pmatrix} (0.10, 0, 0.15) \\ (0, 0, 0.5) \\ (0.3, 0.1, 0.4) \\ (0, 0, 0.3) \end{pmatrix}, \quad G(s_c) = \begin{pmatrix} (0, 0, 0.2) \\ (0.3, 0.4, 0.7) \\ (0, 0, 0.1) \\ (0.3, 0.2, 0.4) \end{pmatrix},$$

$$G(s_s) = \begin{pmatrix} (0.5, 0.1, 0.6) \\ (0.2, 0, 0.3) \\ (0, 0, 0.1) \\ (0.3, 0.2, 0.4) \end{pmatrix}$$

The triangular fuzzy number matrix (G, S) is another parameterized family of triangular fuzzy number matrix and it gives a collection of approximate description of the patients-symptoms in the hospital. Thus, the triangular fuzzy number matrix (G, S) represent the relation matrix $\tilde{R}_S$, where $\tilde{R}_S$ is called the patients-symptoms matrix given by

$$\tilde{R}_S = \begin{matrix} & s_t & s_h & s_c & s_s \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{pmatrix} (0.35, 0.20, 0.45) & (0.10, 0, 0.15) & (0, 0, 0.2) & (0.5, 0.1, 0.6) \\ (0.40, 0.30, 0.45) & (0, 0, 0.5) & (0.3, 0.4, 0.7) & (0.2, 0, 0.3) \\ (0.25, 0.30, 0.4) & (0.3, 0.1, 0.4) & (0, 0, 0.1) & (0, 0, 0.1) \\ (0.20, 0.25, 0.50) & (0, 0, 0.3) & (0.1, 0, 0.2) & (0.3, 0.2, 0.4) \end{pmatrix} \end{matrix}$$

and also, we take $M = \{med_1, med_2\}$ as universal set, where med_1 and med_2 , represent medication respectively. Suppose that; $T(med_1) = ((0.8, 0.7, 0.9) \quad (0.7, 0.5, 0.8))$, $T(med_2) = ((0.7, 0.6, 0.9) \quad (0.8, 0.6, 0.9))$.

The triangular fuzzy number matrix (T, M) is another parameterized family of triangular fuzzy number matrix and it gives a collection of approximate description of the medication-symptoms in the hospital. Thus, the triangular fuzzy number matrix (T, M) represent the relation matrix R_m , where R_m is called the medication-symptoms matrix given by

$$\tilde{R}_m = \begin{matrix} & s_t & & s_c \\ med_1 & ((0.8, 0.7, 0.9) & (0.7, 0.5, 0.8)) \\ med_2 & ((0.7, 0.6, 0.9) & (0.8, 0.6, 0.9)) \end{matrix}$$

Suppose that the parameterized triangular fuzzy number for recovering state of the patients is:

$$\tilde{R}_0^* = \begin{matrix} & d_v & & & & s_t \\ s_t & ((0.60, 0.30, 0.50)) \\ s_h & ((0.15, 0.5, 0.20)) \\ s_c & ((0.35, 0.15, 0.30)) \\ s_c & ((0.20, 0.15, 0.25)) \end{matrix}, \quad \tilde{R}_m^* = \begin{matrix} & & & s_t \\ med_1 & ((0.5, 0.4, 0.6)) \\ med_2 & ((0.5, 0.3, 0.7)) \end{matrix}$$

$$\tilde{R}_s^* = \begin{matrix} & s_t & s_h & s_c & s_s \\ p_1 & ((0.25, 0.10, 0.30) & (0.5, 0, 0.9) & (0, 0, 0) & (0.3, 0, 0.4)) \\ p_2 & ((0.20, 0.15, 0.25) & (0, 0, 0.2) & (0, 0, 0) & (0.1, 0, 0.2)) \\ p_3 & ((0.10, 0.5, 0.20) & (0, 0, 0.1) & (0, 0, 0) & (0, 0, 0.1)) \\ p_4 & ((0.15, 0.10, 0.20) & (0, 0, 0.2) & (0, 0, 0) & (0.1, 0.1, 0.2)) \end{matrix}$$

Where,

$\tilde{R}_s^*$:- patients – symptoms matrix (recovering state), $\tilde{R}_m^*$:- medication – reduced symptoms matrix, $\tilde{R}_0^*$: symptoms- reduced disease matrix.

We consider the pair fully fuzzy matrix equation below:

$$\begin{aligned} \tilde{R}_s \tilde{X} + \tilde{X} \tilde{R}_m &= \tilde{R}_0 \\ \tilde{R}_s^* \tilde{X} \tilde{R}_m^* &= \tilde{R}_0^* \end{aligned} \quad (4.3.1)$$

where,

$$\tilde{R}_s = \begin{pmatrix} (0.35, 0.20, 0.45) & (0.10, 0, 0.15) & (0, 0, 0.2) & (0.5, 0.1, 0.6) \\ (0.40, 0.30, 0.45) & (0, 0, 0.5) & (0.3, 0.4, 0.7) & (0.2, 0, 0.3) \\ (0.25, 0.30, 0.4) & (0.3, 0.1, 0.4) & (0, 0, 0.1) & (0, 0, 0.1) \\ (0.20, 0.25, 0.50) & (0, 0, 0.3) & (0.1, 0, 0.2) & (0.3, 0.2, 0.4) \end{pmatrix}$$

$$\tilde{R}_s^* = \begin{pmatrix} (0.25, 0.10, 0.30) & (0.5, 0, 0.9) & (0, 0, 0) & (0.3, 0, 0.4) \\ (0.20, 0.15, 0.25) & (0, 0, 0.2) & (0, 0, 0) & (0.1, 0, 0.2) \\ (0.10, 0.5, 0.20) & (0, 0, 0.1) & (0, 0, 0) & (0, 0, 0.1) \\ (0.15, 0.10, 0.20) & (0, 0, 0.2) & (0, 0, 0) & (0.1, 0.1, 0.2) \end{pmatrix}$$

$$\tilde{R}_m = \begin{pmatrix} (0.8, 0.7, 0.9) & (0.7, 0.5, 0.8) \\ (0.7, 0.6, 0.9) & (0.8, 0.6, 0.9) \end{pmatrix}, \quad \tilde{R}_m^* = \begin{pmatrix} (0.5, 0.4, 0.6) \\ (0.5, 0.3, 0.7) \end{pmatrix},$$

$$\tilde{R}_0 = \begin{pmatrix} (0.75, 0.50, 0.80) & (0.75, 0.60, 1.0) \\ (0.25, 0.20, 0.30) & (0.50, 0.35, 1.0) \\ (0.55, 0.45, 0.60) & (0.75, 0.70, 1.0) \\ (0.30, 0.25, 0.40) & (0.45, 0.75, 1.0) \end{pmatrix}, \quad \tilde{R}_0^* = \begin{pmatrix} (0.60, 0.30, 0.50) \\ (0.15, 0.5, 0.20) \\ (0.35, 0.15, 0.30) \\ (0.20, 0.15, 0.25) \end{pmatrix}, \text{ and}$$

$$\tilde{X} = \begin{pmatrix} (m_{11}^x, \alpha_{11}^x, \beta_{11}^x) & (m_{12}^x, \alpha_{12}^x, \beta_{12}^x) \\ (m_{21}^x, \alpha_{21}^x, \beta_{21}^x) & (m_{22}^x, \alpha_{22}^x, \beta_{22}^x) \\ (m_{31}^x, \alpha_{31}^x, \beta_{31}^x) & (m_{32}^x, \alpha_{32}^x, \beta_{32}^x) \\ (m_{41}^x, \alpha_{41}^x, \beta_{41}^x) & (m_{42}^x, \alpha_{42}^x, \beta_{42}^x) \end{pmatrix} \geq 0$$

Solution

Considering the following pair of fully fuzzy matrix equation, $\tilde{R}_S = (\tilde{r}_S)_{m \times m}$, $\tilde{R}_S^* = (\tilde{r}_S^*)_{m \times m}$, $\tilde{R}_m = (\tilde{r}_m)_{m \times m}$, $\tilde{R}_m^* = (\tilde{r}_m^*)_{m \times n}$ and $\tilde{R}_0 = (\tilde{r}_0)_{m \times n}$, $\tilde{R}_0^* = (\tilde{r}_0^*)_{m \times n}$. $\tilde{U}$ is fuzzy unitary matrix.

3.1:- The pair of fully fuzzy matrix equation is converted to fully fuzzy linear system by taking the vec operator of both sides $\tilde{R}_S \tilde{X} + \tilde{X} \tilde{R}_m$ and $\tilde{R}_0$ we have

From theorem 4, $\{(\tilde{U}_2 \otimes_k \tilde{R}_S) + (\tilde{R}_m^T \otimes_k \tilde{U}_4)\} \text{Vec}(\tilde{X}) = \text{Vec}(\tilde{R}_0)$ (18)

$$\begin{pmatrix} (0.15, 0.9, 1.35) & (0.10, 0, 0.15) & (0, 0, 0.2) & (0.5, 0.1, 0.6) & (0.7, 0.6, 0.9) & (0, 0, 0) & (0, 0, 0) & (0, 0, 0) \\ (0.40, 0.30, 0.45) & (0.8, 0.7, 1.4) & (0.3, 0.4, 0.7) & (0.2, 0, 0.3) & (0, 0, 0) & (0.7, 0.6, 0.9) & (0, 0, 0) & (0, 0, 0) \\ (0.25, 0.30, 0.15) & (0.3, 0.1, 0.4) & (0.8, 0.7, 1) & (0, 0, 0.1) & (0, 0, 0) & (0, 0, 0) & (0.7, 0.6, 0.9) & (0, 0, 0) \\ (0.20, 0.25, 0.50) & (0, 0, 0.3) & (0.1, 0, 0.2) & (1.1, 0.9, 1.3) & (0, 0, 0) & (0, 0, 0) & (0, 0, 0) & (0.7, 0.6, 0.9) \\ (0.7, 0.5, 0.8) & (0, 0, 0) & (0, 0, 0) & (0, 0, 0) & (0.15, 0.8, 1.35) & (0.10, 0, 0.15) & (0, 0, 0.2) & (0.5, 0.1, 0.6) \\ (0, 0, 0) & (0.7, 0.5, 0.8) & (0, 0, 0) & (0, 0, 0) & (0.40, 0.30, 0.45) & (0.8, 0.6, 1.4) & (0.3, 0.4, 0.7) & (0.2, 0, 0.3) \\ (0, 0, 0) & (0, 0, 0) & (0.7, 0.5, 0.8) & (0, 0, 0) & (0.25, 0.30, 0.15) & (0.3, 0.1, 0.4) & (0.8, 0.6, 1) & (0, 0, 0.1) \\ (0, 0, 0) & (0, 0, 0) & (0, 0, 0) & (0.7, 0.5, 0.8) & (0.20, 0.25, 0.50) & (0, 0, 0.3) & (0.1, 0, 0.2) & (1.1, 0.8, 1.3) \end{pmatrix} \begin{pmatrix} (m_{11}^x, \alpha_{11}^x, \beta_{11}^x) \\ (m_{21}^x, \alpha_{21}^x, \beta_{21}^x) \\ (m_{31}^x, \alpha_{31}^x, \beta_{31}^x) \\ (m_{41}^x, \alpha_{41}^x, \beta_{41}^x) \\ (m_{12}^x, \alpha_{12}^x, \beta_{12}^x) \\ (m_{22}^x, \alpha_{22}^x, \beta_{22}^x) \\ (m_{32}^x, \alpha_{32}^x, \beta_{32}^x) \\ (m_{42}^x, \alpha_{42}^x, \beta_{42}^x) \end{pmatrix} = \begin{pmatrix} (0.75, 0.50, 0.80) \\ (0.25, 0.20, 0.30) \\ (0.55, 0.45, 0.60) \\ (0.30, 0.25, 0.40) \\ (0.75, 0.60, 1.0) \\ (0.50, 0.35, 1.0) \\ (0.75, 0.70, 1.0) \\ (0.45, 0.75, 1.0) \end{pmatrix}$$

and also, by taking the vec operator of both sides $\tilde{R}_S^* \tilde{X} \tilde{R}_m^* = \tilde{R}_0^*$, we have

From theorem 6, $(\tilde{R}_m^{*T} \otimes_k \tilde{R}_S^*) \text{Vec}(\tilde{X}) = \text{Vec}(\tilde{R}_0^*)$ (19)

$$\begin{pmatrix} (0.13, 0.15, 0.3) & (0.25, 0, 0.75) & (0, 0, 0) & (0.15, 0.12, 0.38) & (0.13, 0.103, 0.33) & (0.25, 0.8, 0.07) & (0, 0, 0) & (0.15, 0.09, 0.29) \\ (0.1, 0.14, 0.25) & (0, 0, 0.1) & (0, 0, 0) & (0.05, 0.04, 0.16) & (0.28, 0.14, 0.27) & (0, 0, 0.1) & (0, 0, 0) & (0.05, 0.03, 0.17) \\ (0.05, 0.5, 0.29) & (0, 0, 0.05) & (0, 0, 0) & (0.05, 0.29, 0.16) & (0.05, 0.28, 0.17) & (0, 0, 0.05) & (0, 0, 0) & (0, 0, 0.05) \\ (0.08, 0.14, 0.19) & (0, 0, 0.1) & (0, 0, 0) & (0.05, 0.09, 0.16) & (0.08, 0.08, 0.21) & (0, 0, 0.1) & (0, 0, 0) & (0.05, 0.08, 0.17) \end{pmatrix} \begin{pmatrix} (m_{11}^x, \alpha_{11}^x, \beta_{11}^x) \\ (m_{21}^x, \alpha_{21}^x, \beta_{21}^x) \\ (m_{31}^x, \alpha_{31}^x, \beta_{31}^x) \\ (m_{41}^x, \alpha_{41}^x, \beta_{41}^x) \\ (m_{12}^x, \alpha_{12}^x, \beta_{12}^x) \\ (m_{22}^x, \alpha_{22}^x, \beta_{22}^x) \\ (m_{32}^x, \alpha_{32}^x, \beta_{32}^x) \\ (m_{42}^x, \alpha_{42}^x, \beta_{42}^x) \end{pmatrix} = \begin{pmatrix} (0.60, 0.30, 0.50) \\ (0.15, 0.5, 0.20) \\ (0.35, 0.15, 0.30) \\ (0.20, 0.15, 0.25) \end{pmatrix}$$

Hence the PFFME is transferred to the following PFFLS.

Step 2:- We now convert the pair fully fuzzy linear system to associated linear system.

In order to achieve an associated linear system, all matrices obtained need to be converted to crisp form of matrices. Then, Eq. (18) yields:

$$F_1 = \begin{pmatrix} 0.15 & 0.10 & 0 & 0.5 & 0.7 & 0 & 0 & 0 \\ 0.40 & 0.8 & 0.3 & 0.2 & 0 & 0.7 & 0 & 0 \\ 0.25 & 0.3 & 0.8 & 0 & 0 & 0 & 0.7 & 0 \\ 0.20 & 0 & 0.1 & 1.1 & 0 & 0 & 0 & 0.7 \\ 0.7 & 0 & 0 & 0 & 0.15 & 0.10 & 0 & 0.5 \\ 0 & 0.7 & 0 & 0 & 0.40 & 0.8 & 0.3 & 0.2 \\ 0 & 0 & 0.7 & 0.7 & 0.25 & 0.3 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0.20 & 0 & 0.1 & 1.0 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 0.9 & 0 & 0 & 0.1 & 0.6 & 0 & 0 & 0 \\ 0.30 & 0.7 & 0.4 & 0 & 0 & 0.6 & 0 & 0 \\ 0.30 & 0.1 & 0.7 & 0 & 0 & 0 & 0.6 & 0 \\ 0.25 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0.6 \\ 0.5 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0.1 \\ 0 & 0.5 & 0 & 0 & 0.30 & 0.6 & 0.4 & 0 \\ 0 & 0 & 0.5 & 0 & 0.30 & 0.1 & 0.6 & 0 \\ 0 & 0 & 0 & 0.5 & 0.25 & 0 & 0 & 0.8 \end{pmatrix},$$

$$N_1 = \begin{pmatrix} 1.35 & 0.15 & 0.2 & 0.6 & 0.9 & 0 & 0 & 0 \\ 0.45 & 1.4 & 0.7 & 0.3 & 0 & 0.9 & 0 & 0 \\ 0.15 & 0.4 & 1 & 0.1 & 0 & 0 & 0.9 & 0 \\ 0.50 & 0.3 & 0.2 & 1.3 & 0 & 0 & 0 & 0.9 \\ 0.8 & 0 & 0 & 0 & 1.35 & 0.15 & 0.2 & 0.6 \\ 0 & 0.8 & 0 & 0 & 0.45 & 1.4 & 0.7 & 0.3 \\ 0 & 0 & 0.8 & 0 & 0.15 & 0.4 & 1 & 0.1 \\ 0 & 0 & 0 & 0.8 & 0.50 & 0.3 & 0.2 & 1.3 \end{pmatrix}$$

and $\text{Vec}(\tilde{R}_0)$ yields

$$m^{c_1} = \begin{pmatrix} 0.75 \\ 0.25 \\ 0.55 \\ 0.30 \\ 0.75 \\ 0.50 \\ 0.75 \\ 0.45 \end{pmatrix}, \quad \alpha^{c_1} = \begin{pmatrix} 0.50 \\ 0.20 \\ 0.45 \\ 0.25 \\ 0.60 \\ 0.35 \\ 0.70 \\ 0.75 \end{pmatrix}, \quad \beta^{c_1} = \begin{pmatrix} 0.80 \\ 0.30 \\ 0.60 \\ 0.40 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{pmatrix}$$

On the other hand, the crisp matrices extracted from $\tilde{R}_m^{*T} \otimes_k \tilde{R}_S^*$, Eq. (19) yields

$$F_2 = \begin{pmatrix} 0.13 & 0.25 & 0 & 0.15 & 0.13 & 0.25 & 0 & 0.15 \\ 0.1 & 0 & 0 & 0.05 & 0.28 & 0 & 0 & 0.05 \\ 0.05 & 0 & 0 & 0.05 & 0.05 & 0 & 0 & 0 \\ 0.08 & 0 & 0 & 0.05 & 0.08 & 0 & 0 & 0.05 \end{pmatrix},$$

$$M_2 = \begin{pmatrix} 0.13 & 0.25 & 0 & 0.15 & 0.13 & 0.25 & 0 & 0.15 \\ 0.1 & 0 & 0 & 0.05 & 0.28 & 0 & 0 & 0.05 \\ 0.05 & 0 & 0 & 0.05 & 0.05 & 0 & 0 & 0 \\ 0.08 & 0 & 0 & 0.05 & 0.08 & 0 & 0 & 0.05 \end{pmatrix}$$

$$N_2 = \begin{pmatrix} 0.13 & 0.25 & 0 & 0.15 & 0.13 & 0.25 & 0 & 0.15 \\ 0.1 & 0 & 0 & 0.05 & 0.28 & 0 & 0 & 0.05 \\ 0.05 & 0 & 0 & 0.05 & 0.05 & 0 & 0 & 0 \\ 0.08 & 0 & 0 & 0.05 & 0.08 & 0 & 0 & 0.05 \end{pmatrix}$$

and $\text{Vec}(\tilde{R}_0^*)$ yields

$$m^{c_2} = \begin{pmatrix} 0.60 \\ 0.15 \\ 0.35 \\ 0.20 \end{pmatrix}, \quad \alpha^{c_2} = \begin{pmatrix} 0.30 \\ 0.5 \\ 0.15 \\ 0.15 \end{pmatrix}, \quad \beta^{c_2} = \begin{pmatrix} 0.50 \\ 0.20 \\ 0.30 \\ 0.25 \end{pmatrix}$$

Thus, the crisp matrix is composed to an associated linear system as follows

Composing crisp matrix to an associated linear system (from mathematical model in medical diagnosis)

0.15	0.10	0	0.5	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.40	0.8	0.3	0.2	0	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0
0.25	0.30	0.8	0	0	0	0.7	0	0	0	0	0	0	0	0	0	0	0	0
0.20	0	0.1	1.1	0	0	0	0.7	0	0	0	0	0	0	0	0	0	0	0
0.7	0	0	0	0.15	0.10	0	0.5	0	0	0	0	0	0	0	0	0	0	0
0	0.7	0	0	0.40	0.8	0.3	0.2	0	0	0	0	0	0	0	0	0	0	0
0	0	0.7	0.7	0.25	0.3	0.8	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.20	0	0.1	1.1	0	0	0	0	0	0	0	0	0	0	0
0.13	0.25	0	0.15	0.13	0.25	0	0.15	0	0	0	0	0	0	0	0	0	0	0
0.1	0	0	0.05	0.28	0	0	0.05	0	0	0	0	0	0	0	0	0	0	0
0.05	0	0	0.05	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.08	0	0	0.05	0.08	0	0	0.05	0	0	0	0	0	0	0	0	0	0	0
0.9	0	0	0.1	0.6	0	0	0	0.15	0.10	0	0.5	0.7	0	0	0	0	0	0
0.30	0.7	0.4	0	0	0.6	0	0	0.40	0.8	0.8	0.2	0	0.7	0	0	0	0	0
0.30	0.1	0.7	0	0	0	0.6	0	0.25	0.3	0.3	0	0	0	0.7	0	0	0	0
0.25	0	0	0.9	0	0	0	0.6	0.20	0	0	1.1	0	0	0	0.7	0	0	0
0.5	0	0	0	0.8	0	0	0.1	0.7	0	0	0	0.15	0.10	0	0.5	0	0	0
0	0.5	0	0	0.30	0.6	0.4	0	0	0.7	0.7	0	0.40	0.8	0.3	0.2	0	0	0
0	0	0.5	0	0.30	0.1	0.6	0	0	0	0	0.7	0.25	0.3	0.8	0	0	0	0
0	0	0	0.5	0.25	0	0	0.8	0	0	0	0	0.20	0	0.1	1.1	0	0	0
0.13	0.25	0	0.15	0.13	0.25	0	0.15	0.13	0.25	0.25	0.15	0.13	0.25	0	0.15	0	0	0
0.1	0	0	0.05	0.28	0	0	0.05	0.1	0	0	0.05	0.28	0	0	0.05	0	0	0
0.05	0	0	0.05	0.5	0	0	0	0.05	0	0	0.05	0.05	0	0	0	0	0	0
0.08	0	0	0.05	0.8	0	0	0.05	0.08	0	0	0.05	0.08	0	0	0.05	0	0	0
1.35	0.15	0.2	0.6	0.9	0	0	0	0	0	0	0	0	0	0	0	0.15	0.10	0
0.45	1.4	0.7	0.3	0	0.9	0	0	0	0	0	0	0	0	0	0	0.40	0.8	0.3
0.15	0.4	1	0.1	0	0	0.9	0	0	0	0	0	0	0	0	0	0.25	0.3	0.8
0.50	0.3	0.2	1.3	0	0	0	0.9	0	0	0	0	0	0	0	0	0.20	0	0.1
0.8	0	0	0	1.35	0.15	0.2	0.6	0	0	0	0	0	0	0	0	0.7	0	0
0	0.8	0	0	0.45	1.4	0.7	0.3	0	0	0	0	0	0	0	0	0.00	0.7	0
0	0	0.8	0	0.15	0.4	1	0.1	0	0	0	0	0	0	0	0	0	0	0.7
0	0	0	0.8	0.50	0.3	0.2	1.3	0	0	0	0	0	0	0	0	0	0	0
0.13	0.25	0	0.15	0.13	0.25	0	0.15	0	0	0	0	0	0	0	0	0.13	0.25	0
0.1	0	0	0.05	0.28	0	0	0.05	0	0	0	0	0	0	0	0	0.1	0	0
0.05	0	0	0.05	0.05	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0
0.08	0	0	0.05	0.08	0	0	0.05	0	0	0	0	0	0	0	0	0.08	0	0

$$\begin{pmatrix} m_{11}^x \\ m_{21}^x \\ m_{31}^x \\ m_{41}^x \\ m_{12}^x \\ m_{22}^x \\ m_{32}^x \\ m_{42}^x \\ \alpha_{11}^x \\ \alpha_{21}^x \\ \alpha_{31}^x \\ \alpha_{41}^x \\ \alpha_{12}^x \\ \alpha_{22}^x \\ \alpha_{32}^x \\ \alpha_{42}^x \\ \beta_{11}^x \\ \beta_{21}^x \\ \beta_{31}^x \\ \beta_{41}^x \\ \beta_{12}^x \\ \beta_{22}^x \\ \beta_{32}^x \\ \beta_{42}^x \end{pmatrix} \begin{pmatrix} 0.75 \\ 0.25 \\ 0.55 \\ 0.30 \\ 0.75 \\ 0.50 \\ 0.75 \\ 0.45 \\ 0.60 \\ 0.15 \\ 0.35 \\ 0.20 \\ 0.50 \\ 0.20 \\ 0.45 \\ 0.25 \\ 0.60 \\ 0.35 \\ 0.70 \\ 0.75 \\ 0.30 \\ 0.5 \\ 0.15 \\ 0.15 \\ 0.80 \\ 0.30 \\ 0.60 \\ 0.40 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 0.50 \\ 0.20 \\ 0.30 \\ 0.25 \end{pmatrix}$$

Using proposition 7, Singular Value Decomposition SDV of $F \neq 0$ then SDV of $S \neq 0$. The SVD is used to get the scalar value of a rectangular matrix. Hence the solution is obtained by $X = S^{-1}B$.

$$\left(\begin{array}{c} m_{11}^x \\ m_{21}^x \\ m_{31}^x \\ m_{41}^x \\ m_{12}^x \\ m_{22}^x \\ m_{32}^x \\ m_{42}^x \\ \alpha_{11}^x \\ \alpha_{21}^x \\ \alpha_{31}^x \\ \alpha_{41}^x \\ \alpha_{12}^x \\ \alpha_{22}^x \\ \alpha_{32}^x \\ \alpha_{42}^x \\ \beta_{11}^x \\ \beta_{21}^x \\ \beta_{31}^x \\ \beta_{41}^x \\ \beta_{12}^x \\ \beta_{22}^x \\ \beta_{32}^x \\ \beta_{42}^x \end{array} \right)$$

0.15	0.10	0	0.5	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.40	0.8	0.3	0.2	0	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.25	0.30	0.8	0	0	0	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0
0.20	0	0.1	1.1	0	0	0	0.7	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0	0	0	0.15	0.10	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0
0	0.7	0	0	0.40	0.8	0.3	0.2	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0.7	0.7	0.25	0.3	0.8	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.20	0	0.1	1.1	0	0	0	0	0	0	0	0	0	0	0	0
0.13	0.25	0	0.15	0.13	0.25	0	0.15	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	0	0.05	0.28	0	0	0.05	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0	0	0.05	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.08	0	0	0.05	0.08	0	0	0.05	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0	0	0.1	0.6	0	0	0	0.15	0.10	0	0.5	0.7	0	0	0	0	0	0	0
0.30	0.7	0.4	0	0	0.6	0	0	0.40	0.8	0.8	0.2	0	0.7	0	0	0	0	0	0
0.30	0.1	0.7	0	0	0	0.6	0	0.25	0.3	0.3	0	0	0	0.7	0	0	0	0	0
0.25	0	0	0.9	0	0	0	0.6	0.20	0	0	1.1	0	0	0	0.7	0	0	0	0
0.5	0	0	0	0.8	0	0	0.1	0.7	0	0	0	0.15	0.10	0	0.5	0	0	0	0
0	0.5	0	0	0.30	0.6	0.4	0	0	0.7	0.7	0	0.40	0.8	0.3	0.2	0	0	0	0
0	0	0.5	0	0.30	0.1	0.6	0	0	0	0	0.7	0.25	0.3	0.8	0	0	0	0	0
0	0	0	0.5	0.25	0	0	0.8	0	0	0	0	0.20	0	0.1	1.1	0	0	0	0
0.13	0.25	0	0.15	0.13	0.25	0	0.15	0.13	0.25	0.25	0.15	0.13	0.25	0	0.15	0	0	0	0

0.1	0	0	0.05	0.28	0	0	0.05	0.1	0	0	0.05	0.28	0	0	0.05	0	0	0	0	0	0	0	0
0.05	0	0	0.05	0.5	0	0	0	0.05	0	0	0.05	0.05	0	0	0	0	0	0	0	0	0	0	0
0.08	0	0	0.05	0.8	0	0	0.05	0.08	0	0	0.05	0.08	0	0	0.05	0	0	0	0	0	0	0	0
1.35	0.15	0.2	0.6	0.9	0	0	0	0	0	0	0	0	0	0	0	0.15	0.10	0	0.5	0.7	0	0	0
0.45	1.4	0.7	0.3	0	0.9	0	0	0	0	0	0	0	0	0	0	0.40	0.8	0.3	0.2	0	0.7	0	0
0.15	0.4	1	0.1	0	0	0.9	0	0	0	0	0	0	0	0	0	0.25	0.3	0.8	0	0	0	0.7	0
0.50	0.3	0.2	1.3	0	0	0	0.9	0	0	0	0	0	0	0	0	0.20	0	0.1	1.1	0	0	0	0.7
0.8	0	0	0	1.35	0.15	0.2	0.6	0	0	0	0	0	0	0	0	0.7	0	0	0	0.15	0.10	0	0.5
0	0.8	0	0	0.45	1.4	0.7	0.3	0	0	0	0	0	0	0	0	0.00	0.7	0	0	0.40	0.8	0.3	0.2
0	0	0.8	0	0.15	0.4	1	0.1	0	0	0	0	0	0	0	0	0	0	0.7	0.7	0.25	0.3	0.8	0
0	0	0	0.8	0.50	0.3	0.2	1.3	0	0	0	0	0	0	0	0	0	0	0	0	0.20	0	0.1	1.1
0.13	0.25	0	0.15	0.13	0.25	0	0.15	0	0	0	0	0	0	0	0	0.13	0.25	0	0.15	0.13	0.25	0	0.15
0.1	0	0	0.05	0.28	0	0	0.05	0	0	0	0	0	0	0	0	0.1	0	0	0.05	0.28	0	0	0.05
0.05	0	0	0.05	0.05	0	0	0	0	0	0	0	0	0	0	0	0.05	0	0	0.05	0.05	0	0	0
0.08	0	0	0.05	0.08	0	0	0.05	0	0	0	0	0	0	0	0	0.08	0	0	0.05	0.08	0	0	0.05

0.75
0.25
0.55
0.30
0.75
0.50
0.75
0.45
0.60
0.15
0.35
0.20
0.50
0.20
0.45
0.25
0.60
0.35
0.70
0.75
0.30
0.5
0.15
0.15
0.80
0.30
0.60
0.40
1.0
1.0
1.0
1.0
0.50
0.20
0.30
0.25

Using proposition 7, Singular Value Decomposition SDV of $F \neq 0$ then SDV of $S \neq 0$. Hence the solution is obtained by $X=S^{-1}B$.

Step 4: Solution obtained using pseudoinverse method (from mathematical model in medical diagnosis).

$$\begin{pmatrix} m_{11}^x \\ m_{21}^x \\ m_{31}^x \\ m_{41}^x \\ m_{12}^x \\ m_{22}^x \\ m_{32}^x \\ m_{42}^x \\ \alpha_{11}^x \\ \alpha_{21}^x \\ \alpha_{31}^x \\ \alpha_{41}^x \\ \alpha_{12}^x \\ \alpha_{22}^x \\ \alpha_{32}^x \\ \alpha_{42}^x \\ \beta_{11}^x \\ \beta_{21}^x \\ \beta_{31}^x \\ \beta_{41}^x \\ \beta_{12}^x \\ \beta_{22}^x \\ \beta_{32}^x \\ \beta_{42}^x \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.25 \\ 0.55 \\ 0.30 \\ 0.75 \\ 0.50 \\ 0.75 \\ 0.45 \\ 0.60 \\ 0.15 \\ 0.35 \\ 0.20 \\ 0.50 \\ 0.20 \\ 0.45 \\ 0.25 \\ 0.60 \\ 0.35 \\ 0.70 \\ 0.75 \\ 0.30 \\ 0.5 \\ 0.15 \\ 0.15 \\ 0.80 \\ 0.30 \\ 0.60 \\ 0.40 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 0.50 \\ 0.20 \\ 0.30 \\ 0.25 \end{pmatrix}$$

Thus, the $\text{vec}(\tilde{X})$ is;

$$\text{vec}(\tilde{X}) = \begin{pmatrix} (m_{11}^x, \alpha_{11}^x, \beta_{11}^x) \\ (m_{21}^x, \alpha_{21}^x, \beta_{21}^x) \\ (m_{31}^x, \alpha_{31}^x, \beta_{31}^x) \\ (m_{41}^x, \alpha_{41}^x, \beta_{41}^x) \\ (m_{12}^x, \alpha_{12}^x, \beta_{12}^x) \\ (m_{22}^x, \alpha_{22}^x, \beta_{22}^x) \\ (m_{32}^x, \alpha_{32}^x, \beta_{32}^x) \\ (m_{42}^x, \alpha_{42}^x, \beta_{42}^x) \end{pmatrix} = \begin{pmatrix} (0.32, -0.56, -0.53) \\ (-1.0, 0.47, 0.81) \\ (-0.02, -1.69, 1.41) \\ (0.27, -0.44, 0.04) \\ (0.78, 0.25, -0.52) \\ (1.0, 0.47, -0.36) \\ (-0.06, -0.54, -0.36) \\ (0.18, 0.30, -0.06) \end{pmatrix}$$

Then, the fuzzy matrix solution is,

$$\tilde{X} = \begin{pmatrix} (m_{11}^x, \alpha_{11}^x, \beta_{11}^x) & (m_{12}^x, \alpha_{12}^x, \beta_{12}^x) \\ (m_{21}^x, \alpha_{21}^x, \beta_{21}^x) & (m_{22}^x, \alpha_{22}^x, \beta_{22}^x) \\ (m_{31}^x, \alpha_{31}^x, \beta_{31}^x) & (m_{32}^x, \alpha_{32}^x, \beta_{32}^x) \\ (m_{41}^x, \alpha_{41}^x, \beta_{41}^x) & (m_{42}^x, \alpha_{42}^x, \beta_{42}^x) \end{pmatrix}$$

$$\tilde{X} = \begin{pmatrix} (0.32, -0.56, -0.53) & (0.78, 0.25, -0.52) \\ (-1.0, 0.47, 0.81) & (1.0, 0.47, -0.36) \\ (-0.02, -1.69, 1.41) & (-0.06, -0.54, -0.36) \\ (0.27, -0.44, 0.04) & (0.18, 0.30, -0.06) \end{pmatrix}$$

Thus,

$$\tilde{X} = \begin{matrix} & d_v & d_m \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{pmatrix} (0.32, -0.56, -0.53) & (0.78, 0.25, -0.52) \\ (-1.0, 0.47, 0.81) & (1.0, 0.47, -0.36) \\ (-0.02, -1.69, 1.41) & (-0.06, -0.54, -0.36) \\ (0.27, -0.44, 0.04) & (0.18, 0.30, -0.06) \end{pmatrix} \end{matrix}$$

Interpretation of the Result

The results show the rate at which the malaria fever and viral fever reduced in the patients.

Conclusion

Most of the researchers those who used Sanchez's approach applied max-min composition to solve fully fuzzy matrix equation, but in this paper, we used fuzzy kronecker product, fuzzy vec operator and also associated linear system approach to solve pair fully fuzzy matrix equation in obtaining the solution. Then the final solution was obtained using pseudoinverse method. The algorithm proposed in this paper is simple and effective.

Recommendation

We now turn our attention to comply with future problems that can be done.

1. We solved the pair fully fuzzy matrix equation when the matrices are non singular value decomposition. Therefore, it is suggested to solve pair fully fuzzy matrix equation when the matrix is singular value decomposition.

2. We followed (m, α, β) form for triangular fuzzy numbers, it is suggested to formulate methods that can deal with other forms like (a, b, c) .
3. We implemented pseudoinverse to get the solution, it is suggested to implement any other suitable method.

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