

Volatility Modeling of Revenue Taxes in Nigeria: Utilizing VAR with GARCH Family Models

Baba Gimba Alhassan^{1,2}, Fadhilah Binti Yusof², Siti Mariam Norrulashikin¹, Ibrahim Lawal Kane³

¹Department of Mathematical Sciences, Universiti Teknologi Malaysia (UTM), Malaysia. ²Department of Statistics, School of Applied and Natural Sciences, Federal Polytechnic Bida (FPB) Niger State Nigeria.

³Department of Mathematical Sciences, Faculty of Applied and Natural Sciences, Umaru Musa Yar'adua University Katsina State.

Corresponding author: gazhigun@gmail.com

Abstract

Accurate future values forecast is the Policymakers' tool for tax revenue planning. It's owed to the fact that tax revenue volatility is a valuable measure of uncertainty about the economic environment of a country. This paper applies multivariate time series analysis to the monthly (N Naira) tax revenue data spanning from January 1981 to December 2018 to examine the behavior of tax revenue in Nigeria. To capture the volatility effect in tax revenue data, the paper applies vector autoregression (VAR) and GARCH family models. The study divulges that the tax revenue series displays the empirical consistencies such as non-normality non-stationarity, clustering volatility, and serial correlation that validate the application of the GARCH family methods. The study indicates that tax revenue behavior is usually induced by past data about tax revenue. It's also inferring that previous monthly's volatility in tax revenue can distress the present volatility of tax revenue. In addition, the estimate for asymmetric volatility suggests that positive shocks imply a higher next period conditional variance than negative shocks of the same sign. The main policy implication of these results is that since tax revenue volatility may increase administrative costs and reduce the gains to income revenue, knowledge of tax revenue volatility estimation and forecasting is significant for fiscal policy and tax revenue administrative management.

Keywords: Data, GARCH, Modelling, Tax revenue, Volatility.

Introduction

Breeding an analytic time series model to describe the relationships among highly correlated explanatory variables has been quite challenging due to the difficult nature and the inadequate knowledge of the basic mechanisms that are accountable for the relationship. In some cases, the univariate time series analysis is adequate. While, in some situations, it may be restricted. VAR modeling is regarded as appropriate for the estimation if it does not exhibit significant autocorrelation and heteroscedastic effects which can be captured by the nonlinear multivariate GARCH family models. The multivariate GARCH method is the most suitable model for various interconnected time series with a suitable multi-dimensional model. In particular, the volatility process needs some information from the positive semi-definite matrices and multivariate GARCH can correspond to covariate matrices at each point in time (Boussama et al., 2011). El Hedi Arouri et al. (2015) utilized the VAR-GARCH framework to explore volatility spill-overs amongst stock market and gold prices from 2004 to 2011.

This study tends to develop and evaluate a pioneering multivariate GARCH family model that will take into consideration the volatility stages in financial time series. Among the conservative ways of measuring risk connected with the behavior of financial time series are done thorough studies of the variance. Meanwhile, very few financial kinds of literature emphasize resolving both challenges, heteroscedasticity, and persistence in volatility. Thus, in this study, these two challenges will be tackled utilizing financial (tax revenue) data series. Hence, our focal research questions are hereby summarized as follows: i) Do autocorrelation and heteroscedasticity affect the behavior and characteristics of the financial time series (tax revenue) data? ii) Can the benchmark models VAR treat the autocorrelation and heteroscedasticity incorporating GARCH family models? iii) Does the proposed model reduce the volatility persistency in the residual of financial tax revenue variables? This study is aimed at modelling tax revenue using the proposed model, with the following objectives: To identify the tax revenue series data structure, behavior, and characteristics of the federal inland revenue service (FIRS) of Nigeria. To model the correlation and heteroscedasticity residual time series utilizing the multivariate model. Secondary data were obtained for this study from the National Bureau of Statistics (NBS) and Central Bank of Nigeria (CBN) spanning from 1981 to 2018 for 456 monthly tax revenue in Billions of Nigerian currency (N Naira).

The Literature

The research compared several multivariate volatility models that were frequently adopted in finance literature like BEKK GARCH. The results proved the superiority of VAR-GARCH over other multivariate GARCH model specifications. Silva et al., (2020) utilize a bivariate VAR-GARCH model and developed an index of positive and negative news to measure the impact of press releases. Their results reveal an average spread impact of the news for variables used as proxies for Brazilian asset prices. Ayele et al., (2017) investigated the main macroeconomic variables that influence the variable's volatility (gold prices) and formulate various ARCH family models to assess the volatility of gold prices in the Ethiopian market. Besides, the interest of their study is to find out whether GARCH family models are an improvement over the EWMA-type models in terms of modelling and forecasting price volatility of gold Bouri et al (2018) utilized the VAR Asymmetric-GARCH model to study the relationship amongst conventional investments and Bitcoin visa viz volatility spillovers and returns between four asset classes and the largest cryptocurrency (commodities, currencies, equities, stocks, and bonds). Their results indicate that Bitcoin is considered for stability in the financial system globally by fund managers and investors. Symitsi, & Chalvatzis, (2018) employed an asymmetric multivariate VAR-GARCH model to study spillover effects amongst energy, Bitcoin, and technological companies.

MacDonald et al. (2018) investigated co-movements of volatility and effects of spill-over among the Economies of the Eurozone. Likewise, the GARCH-BEKK technique might evaluate the volatility linkage, the returns, and spill-over volatility effects (Kumar, 2013). Ashwin et al., (2010) studied the co-movement amongst some selected world stock markets for both developed and emerging economies utilizing the VAR-fGARCH model. The result indicates average daily positive returns for some selected periods and high volatility of the stock market during the period for the developed stock market and low for emerging markets. The result also reveals that the correlation amongst the stock markets was low for emerging markets. This means that the stock markets have attractive portfolio diversification opportunities for emerging market investors. The original formulation of the GO-GARCH model involved the stronger assumption of independence of the components of y_t , but for the methods presented in the present paper, the conditional uncorrelatedness assumption (Fan

et al. (2008). Multivariate GARCH may be used to indirectly model the correlation among the series compared to directly modelling the variance-covariance matrices.

El Hedi Aroui et al. (2015) utilized the VAR-GARCH framework to explore volatility spill-overs amongst stock market and gold prices from 2004 to 2011. The research compared several multivariate volatility models that were frequently adopted in finance literature like BEKK GARCH. The results proved the superiority of VAR-GARCH over other multivariate GARCH model specifications. Agung et al., (2010) compared non-sharia stock-based and sharia stock-based amongst maximum investment losses due to the distinctive stock index characteristic. Utilizing EWMA, GARCH VAR VAR-EWMA, and VAR-GARCH model. EWMA, GARCH VAR VAR-EWMA, and VAR-GARCH model. The EWMA model performs better and yields a moderate VAR value than the GARCH model likewise VAR-EWMA performs better than the VAR-GARCH model on the characteristic risk of the Capital Market of Indonesia. Tiwari et al., (2020) Compare the combination of ARMA-GARCH univariate models, with VAR-GO-GARCH, VAR-DCC-GARCH, and D-Vine copula ARMA-GARCH multivariate approach VAR-GO-GARCH, VAR-DCC-GARCH, and D-Vine copula ARMA-GARCH multivariate approach in which they conclude that the simple ARMA-GARCH graphically fit well the data for univariate time series modelling. Aue et al (2017). In this article, the theory is developed for a new functional version of the GARCH process, termed fGARCH. The main results are concerned with the structure of the fGARCH(1,1) process, providing criteria for the existence of strictly stationary solutions both in the space of square-integrable and continuous functions.

The Methodology

The VAR-GARCH Model

The VAR-GARCH model incorporates a standard GARCH (1,1) process (Chung et al., 2012,) for the orthogonalized error of each VAR equation:1

$$y_t = B_0 + \sum_{i=1}^p B_i y_{t-i} + v_t$$

$$v_t = A^{-1} H_t^{0.5} \varepsilon_t, \varepsilon_t \sim N(0, I_k), H_t \equiv \text{diag}(h_{1,t}, \dots, h_{k,t}) \quad (1)$$

$$h_{i,t} = a_{0,i} + a_{1,i} v_{i,t-1}^2 + a_{2,i} h_{i,t-1}, a_{0,i} > 0, a_{1,i} + a_{2,i} < 1, i = 1, k$$

where $A = a D$ is a lower triangular matrix with ones on the diagonal and non-zero coefficients below the diagonal. In this GARCH formulation, each variable is treated independently, with the conditional variance $h_{i,t}$ a function of one lag of itself and one lag of the squared error from the VAR equation. We impose conditions to ensure the positivity and stationarity of each volatility process.

The VAR-EWMA Model

Certain multivariate volatility models were set in specifying volatility matrices to be positive and definite. A straightforward technique to appraise time-varying volatilities is the EWMA technique. Once more, let/be the mean equation residuals (Creal et al., 2011). Thus, the volatility for the EWMA technique is expressed as:

$$\hat{\Sigma}_t = \lambda \hat{\Sigma}_{t-1} + (1-\lambda) \hat{a}_{t-1} \hat{a}_{t-1}' \quad (2)$$

where $0 < \lambda < 1$ represents the persistence parameter or rotting rate. If positive definite matrix $\hat{\Sigma}_0$ begins with the recursion, then, the $\hat{\Sigma}_t$ volatility matrix is positive definite for all t . The choice of $\hat{\Sigma}_0$ is obvious for the covariance matrix of the sample $\hat{a}_t$ provided that $T > t$. Hypothetically, the parameter λ might be estimated by the QMLE or secure a priori. Little experience shows that $\hat{\lambda} \approx 0.96$ for several asset return sequences (Clare et al 2016). EWMA is a technique that presumes value changes over time. Utilizing EWMA for volatility modelling, the expression is written as:

$$\sigma = \sqrt{(1-\lambda) \sum_{t=n}^{t=i} \lambda^t (X_t - \mu)^2} \quad (3)$$

Where n is the number of months and λ is an exponential factor. In expression, μ is the μ value of the distribution, which is usually assumed to be zero. The expression of volatility for exponential weighted is expressed as:

$$\sigma_{i,j} = \sqrt{(1-\lambda) \sum_{j=0}^{\infty} \lambda^j r_{i-j}^2} \quad (4)$$

This expression is directly measured up with the GARCH technique. The vital aspect of the ability of the technique is the selected value factor. For EWMA estimate, the required number of days/months can be estimated as follows (Best, 1999)

In equation λ is an exponential factor, $r_{i,j}$, denotes the return of asset logarithmic i at time t . Hence, $r_{i,j}$, is estimated by the $\ln(P_{i,t} / P_{i,t-1})$ formula.

The VAR-GO-GARCH Model

In a multivariate GARCH setting, the conditional covariance matrix of the m -dimensional zero-mean random variable depends on elements of the information set up to time $t-1$, denoted by Σ_{t-1} . Van DerWeide (2002) applies the idea of ICA to offer generalized orthogonal GARCH classes (Go-GARCH) models for modeling volatility. The Go-GARCH model's main hypothesis is to transform matrix M , for the time-invariant (Van DerWeide (2002). the matrix volatility under the hypothesis Σ_t is expressed as: $\Sigma_t = MVM'$, (5)

Where the volatility matrix of b_t is MV , that is, $V_t = \text{Cov}(b_t | F_{t-1})$ with F_{t-1} representing $t-1$ of the information available at the period. The Go-GARCH model's additional main assumption is that for all t is a diagonal matrix of V_t . Considering the k -variate VAR technique of order one $I(1)$ for the mean equation with the representation as:

$$y_t = \mu + By_{t-1} + \varepsilon_t \quad (6)$$

where $\text{VAR}(\varepsilon_t | y_{t-1}, \dots, y_1) = H_t$, y_t is a $k \times 1$ vector of returns, μ is a $k \times 1$ vector of coefficients, the autoregressive coefficients B is a $k \times k$ matrix, and ε_t is a $k \times 1$ vector of innovations as follows.

$$y_t = [y_{1t}, y_{2t}, \dots, y_{kt}]', \quad \mu = [y_1, y_2, \dots, y_k]'$$

$$B = \begin{bmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1k} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{k1} & \beta_{k2} & \dots & \beta_{kk} \end{bmatrix}, \quad \varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{kt}]'$$

The model is stationary if all values of the z resolving equation (7) are outside of the unit circle.

$$|I_k - B_z| = 0 \quad (7)$$

The mean parameters number of the matrices μ and B is $k(k+1)$. Nevertheless, if B is presumed to be diagonal, the mean parameters number is reduced to $2k$. The expression of the error term ε_t is as follows:

$$\varepsilon_t = H_t^{1/2} \eta_t \quad (8)$$

where η_t is a $k \times 1$ vector with $E(\eta_t) = 0$ and $\text{VAR}(\eta_t) = I$

$$H_t = D_t R_t D_t \quad (9)$$

where $D_t = \text{diag}(h_{1t}^{1/2}, h_{2t}^{1/2}, \dots, h_{kt}^{1/2})$ and R_t is the conditional correlation matrix such that

$$H_t = \text{diag}(h_{1t}^{1/2}, h_{2t}^{1/2}, \dots, h_{kt}^{1/2}) \begin{bmatrix} 1 & \rho_{12t} & \dots & \rho_{1kt} \\ \rho_{12t} & 1 & \dots & \rho_{2kt} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1kt} & \rho_{2kt} & \dots & 1 \end{bmatrix} \text{diag}(h_{1t}^{1/2}, h_{2t}^{1/2}, \dots, h_{kt}^{1/2})$$

$$= \begin{bmatrix} h_{1t} & \rho_{12t} \sqrt{h_{1t} h_{2t}} & \dots & \rho_{1kt} \sqrt{h_{1t} h_{kt}} \\ \rho_{12t} \sqrt{h_{1t} h_{2t}} & h_{2t} & \dots & \rho_{2kt} \sqrt{h_{2t} h_{kt}} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1kt} \sqrt{h_{1t} h_{kt}} & \rho_{2kt} \sqrt{h_{1t} h_{kt}} & \dots & h_{kt} \end{bmatrix}$$

As of the preceding expressions, suppose the positive definite conditional correlation matrix R_t , it is obvious that providing conditional variances h_{it} , are positive for any $i=1, 2, \dots, k$, the conditional variance-covariance matrix, H_t , would be positive definite as well. The conditional variances h_t are presumed to follow a GARCH(1,1) model. Then,

$$h_t = \omega + A\varepsilon_{t-1}^{(2)} + Gh_{t-1} \quad (10)$$

where $h_t = [h_{1t}, h_{2t}, \dots, h_{kt}]'$ and $\varepsilon_t^{(2)} = [\varepsilon_{1t}^2, \varepsilon_{2t}^2, \dots, \varepsilon_{kt}^2]'$ are $k \times 1$ vectors of conditional variances and squared errors correspondingly and ω is a $k \times 1$ and the matrices coefficients of A and

G are $k \times k$. If A and G are constrained towards diagonal, then the spill-overs volatility might not be annexed;

The VAR-fGARCH Model

Definition. A random function $(y_i : i \in \mathcal{E})$ of the series is referred as functional GARCH procedure of orders $(1,1)$, summarised as fGARCH(1,1), if it gratifies the expressions as:

$$y_i = \sigma_i \varepsilon_i, \quad (11)$$

$$\sigma_i^2 = \delta + \alpha y_{i-1}^2 + \beta \sigma_{i-1}^2 \quad (12)$$

Where the map of non-negative functions to the integral operators, and the integral operators to non-negative functions, which notation is short and denoted as:

$$\sigma \geq 0, \alpha \geq 0, \beta \geq 0, \quad (13)$$

We introduced further assumptions where necessary. Remark that the integral operators of α and β are, $t \in [0,1]$, however, the GARCH(1,1) model is unable to completely capture heteroscedasticity fGARCH can capture it better compared to other models of the GARCH family model (Rice, G; et al 2020) The conditional variance of the fGARCH technique in (13) at time $h+1$ is given by

$$\sigma_{h+1}^2 = \alpha_0 + \sum_{i=1}^m \alpha_i a_{h+1-i}^2 + \sum_{j=1}^s \beta_j \sigma_{h+1-j}^2 \quad (14)$$

where at time h all the past values a_{h+1-i}^2 and σ_{h+1-j}^2 are known for all $i \leq m$ and $j \leq s$.

Therefore, from forecast origin h $\sigma_h^2(1)$ equals σ_{h+1}^2 for the 1-step ahead forecast given as:

$$\sigma_h^2(1) = \alpha_0 + \sum_{i=1}^m \alpha_i a_{h+1-i}^2 + \sum_{j=1}^s \beta_j \sigma_{h+1-j}^2 \quad (15)$$

In particular, for a GARCH(1,1) process with $\alpha_0 = 0$, the 1-step-ahead forecast from forecast origin h is

$$\sigma_h^2(1) = \alpha_1 a_h^2 + \beta_1 \sigma_h^2$$

Result Discussion

VAR-GARCH Model

Figure. 1 and 2 gives the time plots of the time-varying correlations between the tax component's returns. The red line denotes the estimated values spiking on the actual values indicating the performance of the combination of the VAR-GARCH model is done well, by removing the autocorrelation, but the underlying assumption for the VAR-GARCH model is that volatility changes

over time. When σ_i^2 is high, there is a tendency σ_{i+1}^2 to be high and vice-versa.

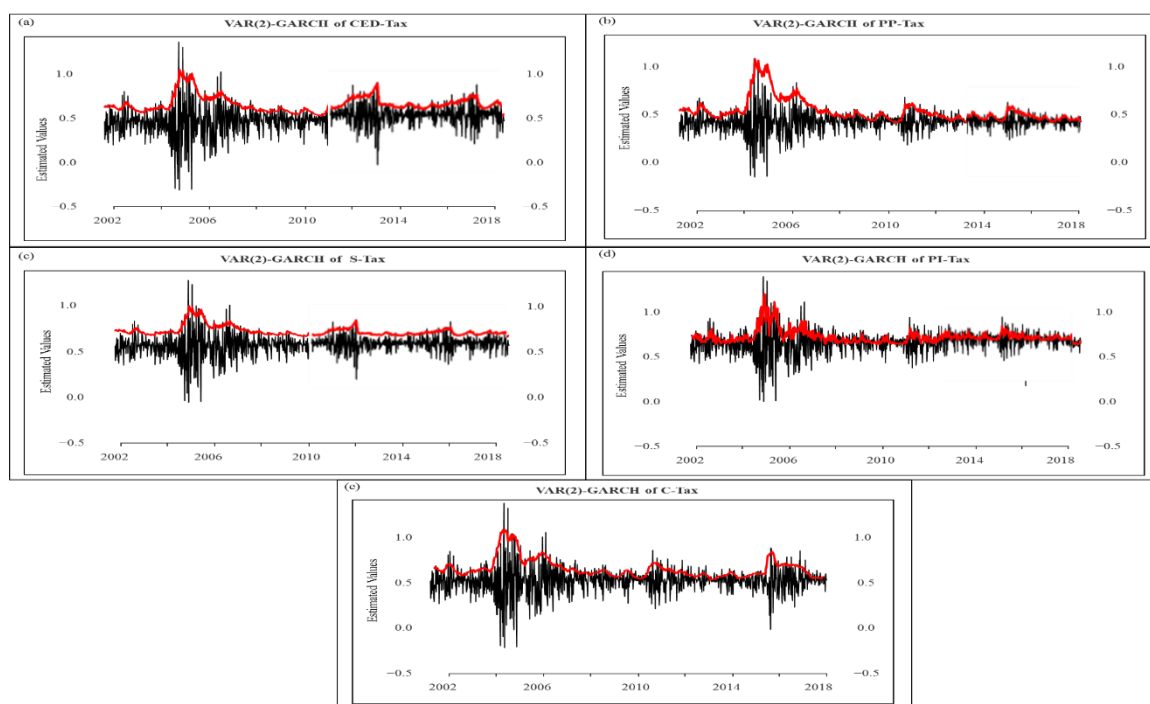


Figure. 1 (a)-(e) Indicating estimated values of the VAR(2)-GARCH model

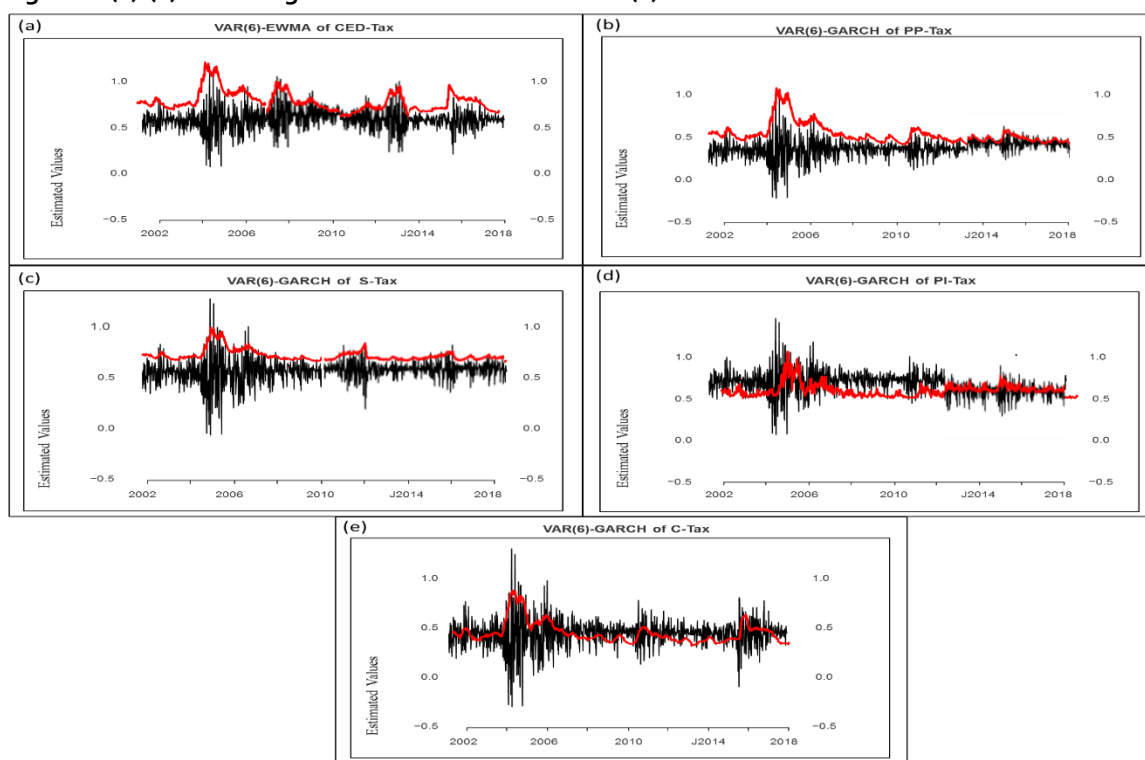


Figure. 2 (a)-(e) Indicating estimated values of the VAR(6)-GARCH model

The result from Table 1 showed that the estimates of α and β are statistically significant with the probability of less than a 5% significance level. The conditional variance was finite and the series was firmly stationary. Though, the sum of parameters α β for the VAR-GARCH model is shown

to approach one. This value indicated that a large long-term influence shock on the conditional correlation is present in the volatility series (Morema & Bonga-Bonga., 2020; Park et al., 2017).

Table 1. VAR-GARCH parameter estimation.

		Estimates	Standard error	Probability
VAR(2)				
Parameter	α	0.0461	0.0124	0.0000
	β	0.9681	0.9792	0.0015
Information Criteria	AIC	29.1497		
	BIC	29.5131		
Log-likelihood		-2874.4111		
VAR(6)				
Parameter	α	0.0572	0.0171	0.0023
	β	0.9591	0.9612	0.0125
Information Criteria	AIC	29.4527		
	BIC	29.7324		
Log-likelihood		-2224.4551		

Table 2 shows the estimated performance of the models in terms of error measurements, that is mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean square error (RMSE) we considered the model with minimum values, such that we can select a model that yields more accurate results.

Table 2 The Accuracy Measures VAR-GARCH Model

Model	MAE	MAPE	RMSE
In-sample			
VAR(2)-Garch	0.5432	22.8923	22.8977
VAR(6)-GARCH	0.5954	22.9856	22.9888
Out-sample			
VAR(2)-GARCH	0.2732	23.4365	23.4367
VAR(6)-GARCH	0.2991	23.8768	23.8798

VAR-EWMA Model

Figure. 3 and 4 give the time plots of the time-varying correlations between the tax component's returns. The red line denotes the estimated values spiking on the actual values indicating the performance of the combination of the VAR-EWMA model has performed better, by eliminating the autocorrelation, but the underlying assumption for the VAR-EWMA model is that volatility changes

over time. When U_i^2 is high, there is a tendency U_{i+1}^2 to be high and vice-versa

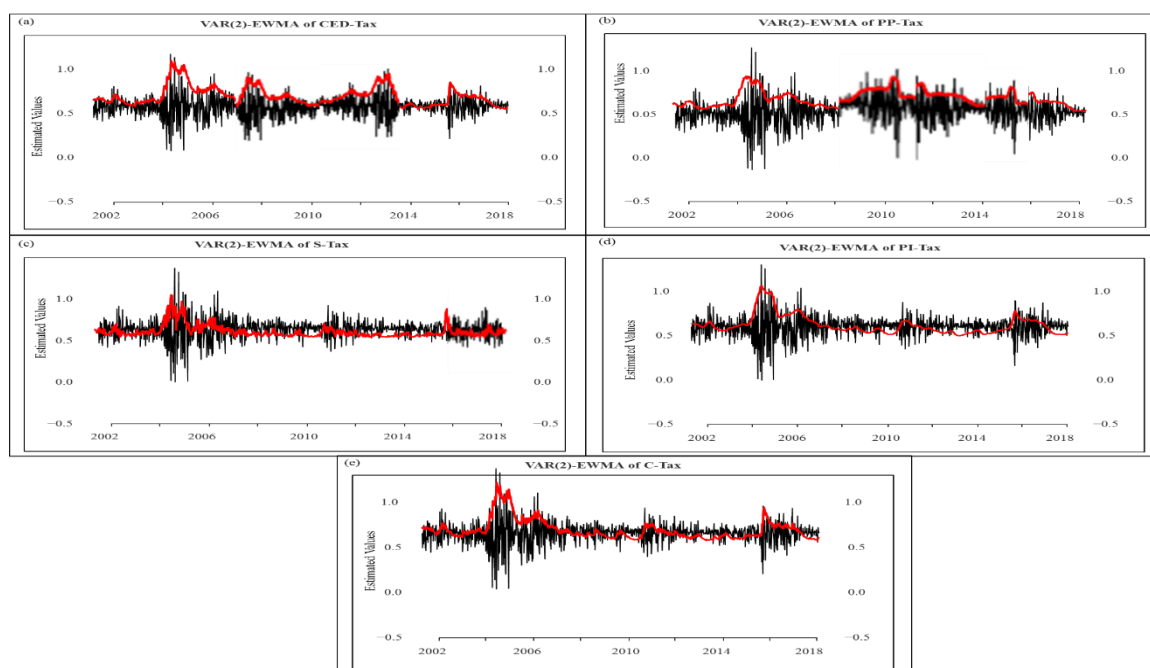


Figure. 3 (a)-(e) indicating estimated values of the VAR(2)-EWMA model

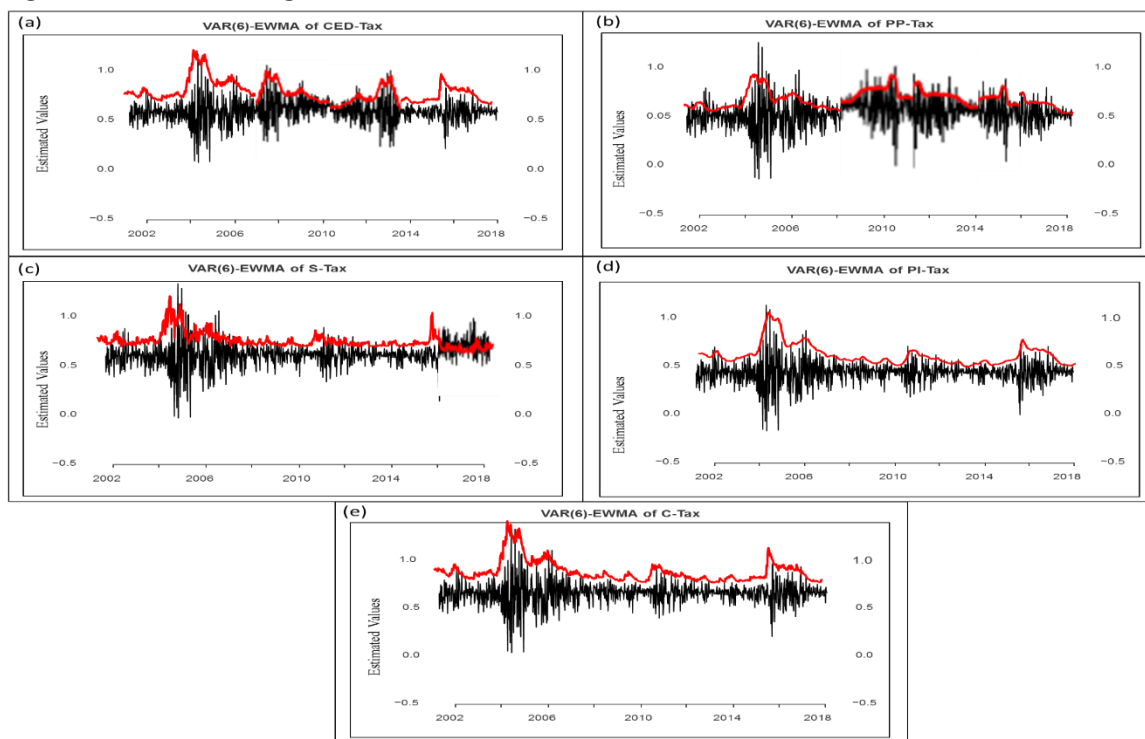


Figure. 4 (a)-(e) indicating estimated values of the VAR(6)-EWMA model

The result from Table 3 showed that the estimates of α and β are statistically significant with the probability of less than a 5% significance level. The conditional variance was finite and the series were strictly stationary. But, the sum of parameters α and β for the VAR-EWMA model is divulged to approach one. This value displayed that a bulky long-term influential shock on the conditional correlation is shown in the volatility series (Halkos & Tzirivis 2019; Zivkov et al., 2020).

Table 3 VAR-EWMA parameter estimation.

		Estimates	Standard error	Probability
VAR(2)-EWMA				
Parameter	α	0.0511	0.0154	0.0000
	β	0.9712	0.9622	0.0021
Information Criteria	AIC	31.1007		
	BIC	31.1018		
Log-likelihood		-2834.4721		
VAR(6)-EWMA				
Parameter	α	0.5221	0.0432	0.0121
	β	0.9621	0.9561	0.0127
Information Criteria	AIC	31.7851		
	BIC	31.7892		
Log-likelihood		-2464.7621		

Table 4 shows the estimated performance of the models in terms of error measurements, that is mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean square error (RMSE) we considered the model with minimum values, such that we can select a model that yields more accurate results.

Table 4 The Accuracy Measures VAR-EWMA Model

Model	MAE	MAPE	RMSE
In-sample			
VAR(2)-EWMA	0.5619	22.2717	28.2928
VAR(6)-EWMA	0.5792	22.3034	22.3075
Out-sample			
VAR(2)-EWMA	0.2422	22.8672	22.8691
VAR(6)-EWMA	0.2244	22.6741	22.6777

VAR-GO-GARCH Model

Figure. 5 and 6 gives the time plots of the time-varying correlations between the tax component's returns. The red line denotes the estimated values spiking on the actual values indicating the performance of the combination of the VAR-GO-GARCH model has performed better, by eradicating the autocorrelation, but the underlying assumption for the VAR-GO-GARCH model is

that, volatility changes over time. When σ_i^2 is high, there is a tendency σ_{i+1}^2 to be high and vice-versa

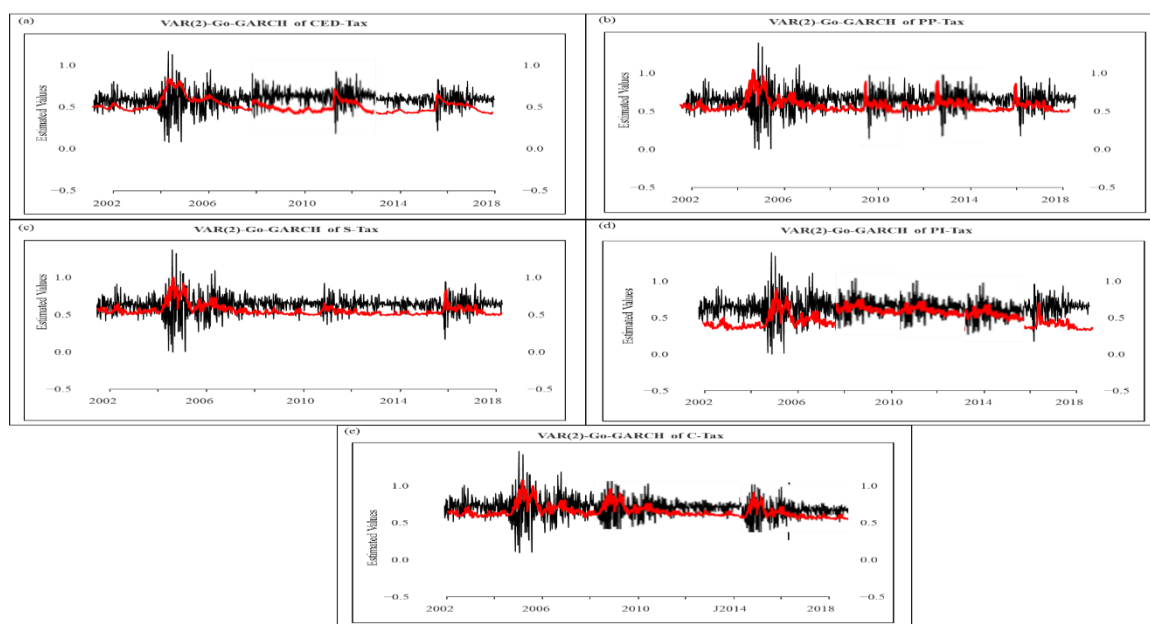


Figure. 5 (a)-(e) indicating estimated values of the VAR(2)-GO-GARCH model

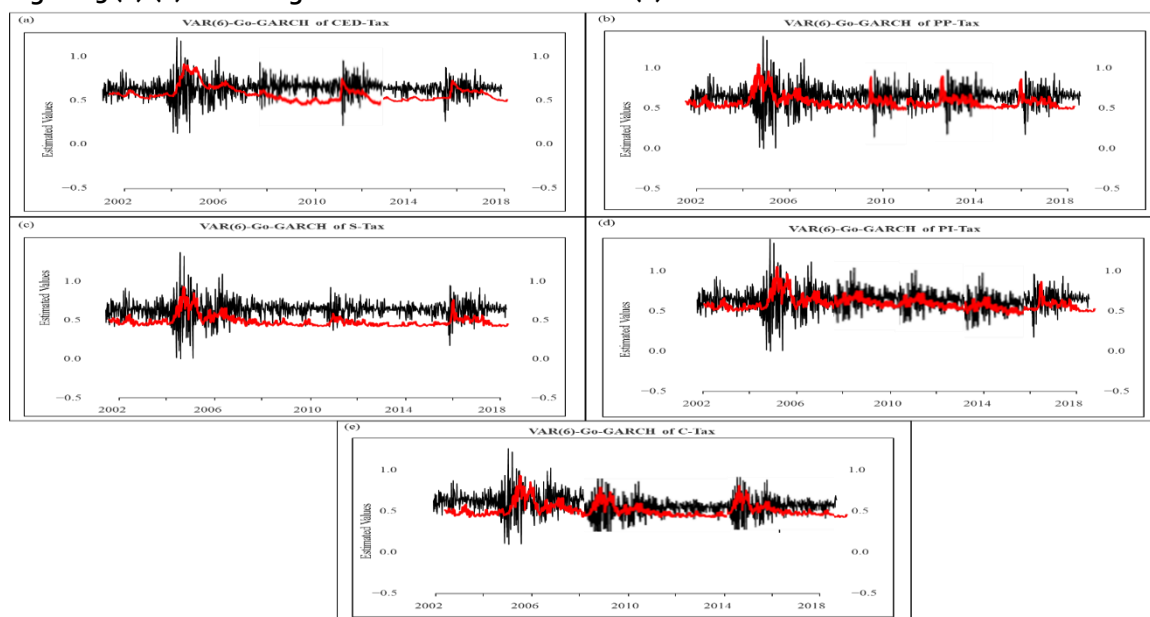


Figure. 6 (a)-(e) indicating estimated values of the VAR(6)-GO-GARCH model

The result from Table 5 exhibited that the estimates of α and β are statistically significant with the probability of less than a 5% significance level. The conditional variance was restricted and the series were stringently stationary. Nevertheless, the sum of parameters α β for the VAR-Go-GARCH model is appeared to approach one. This value indicated that a bulky long-term influence shock on the conditional correlation is portrayed in the volatility series (Sadorsky 2014; Park et al., 2017).

Table 5. VAR-Go-GARCH parameter estimation.

		Estimates	Standard error	Probability
VAR(2)				
Parameter	α	0.0411	0.0114	0.0000
	β	0.9543	0.9122	0.0019
Information Criteria	AIC	29.8327		
	BIC	29.8338		
Log-likelihood		-2894.4121		
VAR(6)				
Parameter	α	0.07711	0.03311	0.0231
	β	0.9543	0.9122	0.0121
Information Criteria	AIC	29.9877		
	BIC	29.9898		
Log-likelihood		-2774.3231		

Table 6 exhibits the evaluated performance of the models in terms of error measurements, that is mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean square error (RMSE) we measured the model with minimum values, such that we can select a model that accedes more precise results.

Table 6 The accuracy measures for the VAR-GO-GARCH model

Model	MAE	MAPE	RMSE
In-sample			
VAR(2)-GO-GARCH	0.4029	21.8973	21.8994
VAR(6)-GO-GARCH	0.4551	21.9932	21.9984
Out-sample			
VAR(2)-GO-GARCH	0.1954	20.9867	20.9898
VAR(6)-GO-GARCH	0.2131	20.9957	20.9991

VAR-fGARCH Model

The fGARCH model was used to model the residuals of the fitted VAR(2) model and the result of the parameter estimation is displayed in Table 10. For this VAR(2)-fGARCH model, parameter a measured the impact of past shocks on conditional correlations while parameter p measured the impact of past correlations. The dynamic part was captured by coefficient α which accounted for the impact of shocks on current conditional correlations (Adammer et al., 2016; Norrulashikin et al., 2018).

Graphical Presentation of VAR-fGARCH Performance

Figure. 7 and 8 gives the time plots of the time-varying correlations between the tax components' returns. The red line represents the estimated values with less spiking on the actual values signifying better performance of the combination of the VAR-fGARCH model, by eliminating the autocorrelation, but the underlying assumption for the VAR-fGARCH model is that volatility changes over time.

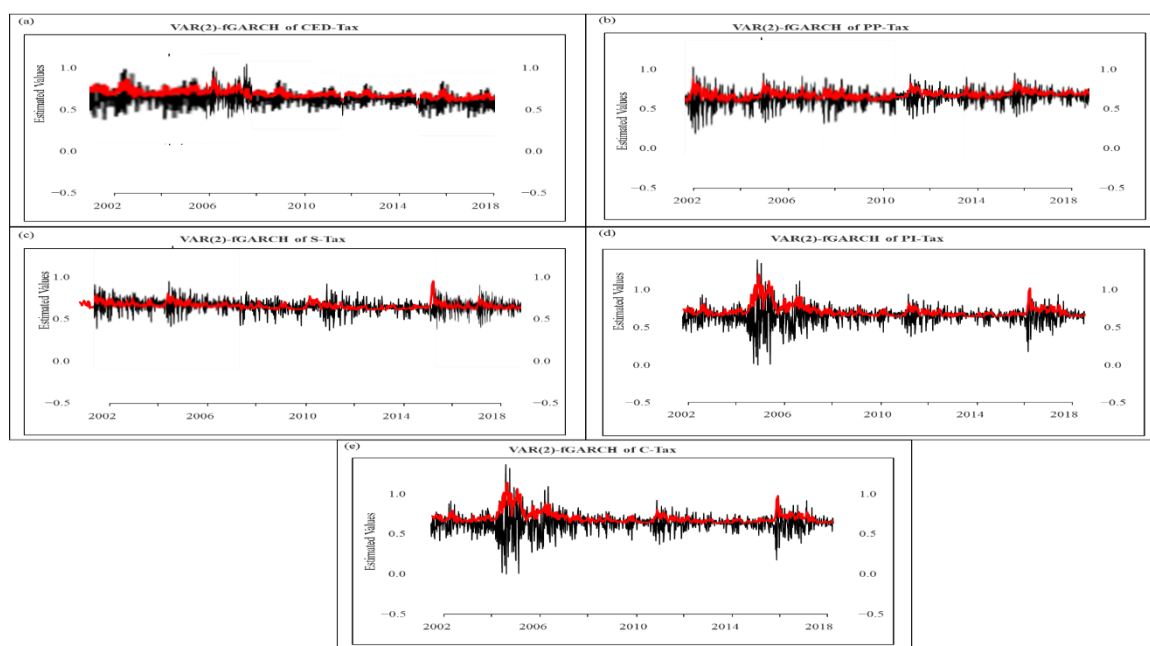


Figure. 7 (a)-(e) Indicating estimated values of the parameter for the VAR(2)-fGARCH model

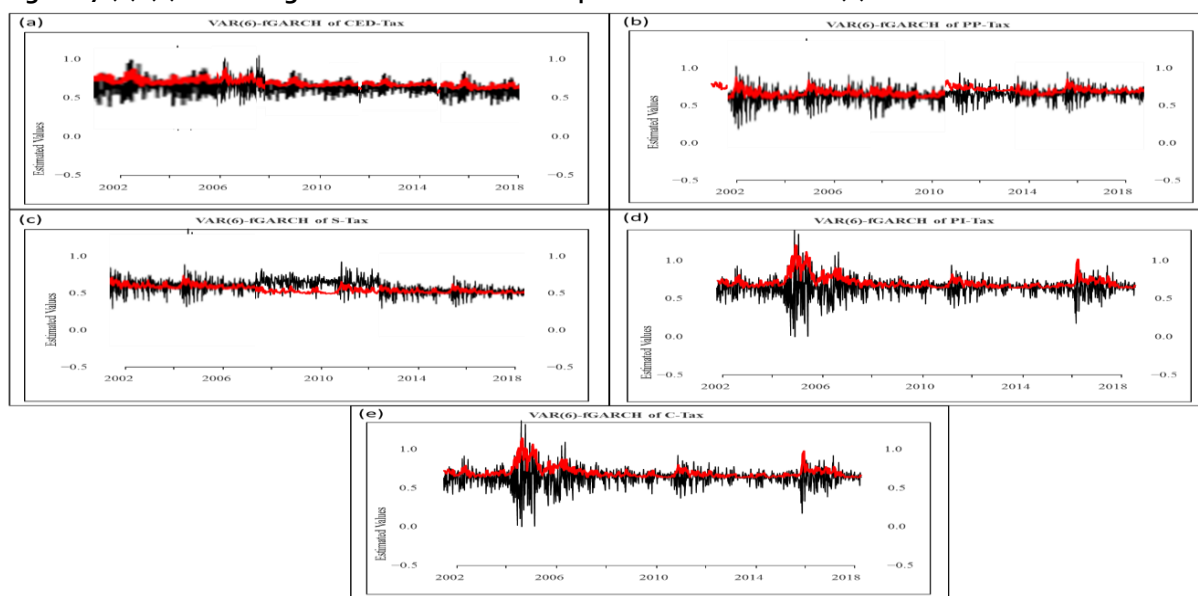


Figure. 8 (a)-(e) Indicating estimated values of the parameter for the VAR(6)-fGARCH model

VAR-fGARCH Model Estimation

The result from Table 7 showed that the estimates of α and β are statistically significant with the probability of less than a 5% significance level. The conditional variance was finite and the series were strictly stationary. However, the sum of parameters α and β for the VAR-fGARCH model is shown to approach one. This value indicated that a large long-term influence shock on the conditional correlation is present in the volatility series (Sadorsky 2014; Park et al., 2017).

Table 7 VAR-fGARCH parameter estimation.

		Estimates	Standard error	Probability
VAR(2)-fGARCH				
Parameter	α	0.0391	0.0094	0.0000
	β	0.9451	0.8992	0.0007
Information Criteria	AIC	28.1157		
	BIC	28.4531		
Log-likelihood		-2974.4221		
VAR(6)-fGARCH				
Parameter	α	0.0411	0.0143	0.0067
	β	0.9393	0.8881	0.0121
Information Criteria	AIC	28.4347		
	BIC	28.4392		
Log-likelihood		-2522.2111		

Table 8 shows the estimated performance of the models in terms of error measurements, that is mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean square error (RMSE) we considered the model with minimum values, such that we can select a model that yields more accurate results.

Table 8 The accuracy measures for the VAR-fGARCH model

Model	MAE	MAPE	RMSE
In-sample			
VAR(2)-fGARCH	0.3944	15.828	20.3997
VAR(6)-fGARCH	0.4345	15.899	20.5455
Out-sample			
VAR(2)-fGARCH	0.1251	11.8767	13.8901
VAR(6)-fGARCH	0.1653	11.9934	13.9789

In other to draw a meaningful conclusion on the model, it is essential to examine whether or not the residuals passed the autocorrelation test, and to measure the adequacy of the model (see Table .9). The autocorrelation test of the residuals from the fitted model was performed using the Breusch-Godfrey test. The residuals seemed to be uncorrelated since the p -value was 0.9629 at more than a 5% significance level. However, the ARCH-LM test with the p -value of 0.0076 which was applied to the squared residuals of the fitted VAR model, had rejected the null hypothesis of no ARCH effect in the residuals. This is because the p -value is less than a 5% significance level. This result indicated the existence of the heteroscedasticity effect in the residual.

Table 9 Residual analysis.

Adequacy checking	Test	Test statistics (p -value)
VAR-GARCH		
Autocorrelation	Breusch-Godfrey	0.3017 (0.1571)
Heteroscedasticity	ARCH-LM	1.143 (0.5615)

VAR-EWMA		
Autocorrelation	Breusch-Godfrey	0.3178 (0.1571)
Heteroscedasticity	ARCH-LM	1.336 (0.5092)
VAR-GO-GARCH		
Autocorrelation	Breusch-Godfrey	0.3117 (1.1474)
Heteroscedasticity	ARCH-LM	1.3434 (0.4655)
VAR(2)-fGARCH		
Autocorrelation	Breusch-Godfrey	0.2927 (0.1154)
Heteroscedasticity	ARCH-LM	1.034 (0.4385)
VAR(6)-GARCH		
Autocorrelation	Breusch-Godfrey	0.4121 (0.1778)
Heteroscedasticity	ARCH-LM	1.187 (0.5453)
VAR(6)-EWMA		
Autocorrelation	Breusch-Godfrey	0.3651 (0.1971)
Heteroscedasticity	ARCH-LM	1.564 (0.5231)
VAR(6)-GO-GARCH		
Autocorrelation	Breusch-Godfrey	0.3436 (0.1871)
Heteroscedasticity	ARCH-LM	1.451 (0.5522)
VAR(6)-fGARCH		
Autocorrelation	Breusch-Godfrey	0.3821 (0.1321)
Heteroscedasticity	ARCH-LM	1.093 (0.5125)

Models Comparison

The comparison of the in-sample and out-sample performances of the VAR-GARCH, VAR-EWMA, VAR-Go-GARCH, and VAR-fGARCH model and VAR-fGARCH model. Table 10 presents the results of cross-validation analysis using MAPE, MAE, and RMSE, respectively. According to Lewis (1982), MAPE percentage error can be categorized into three segments. MAPE error between 20 % to 50 % is in the "reasonable" forecasting power category, 10 % to 20 % in the "good" category, and less than 10 % is in the "excellent" category (Hsu and Wang, 2009). From Table 10, it can be seen that the proposed model, the VAR-GARCH family model's ability to forecast (out-sample) the tax revenue data series falls into the "excellent" category of forecasting power. The smaller values for each in-sample and out-sample are in bold. Meanwhile VAR-fGARCH model yields the minimum error among the models, which is the best model.

Table 10 Cross-validation analysis.

Model	MAPE	MAE	RMSE
In-sample			
VAR(2)-Garch	0.5432	22.8923	22.8977
VAR(2)-EWMA	0.5619	22.2717	28.2928
VAR(2)-GO-GARCH	0.4029	21.8973	21.8974
VAR(2)-fGARCH	0.3944	15.828	20.3997
Out-sample			
VAR(2)-GARCH	0.2732	23.4365	23.4367
VAR(2)-EWMA	0.2422	22.8691	22.8691
VAR(2)-GO-GARCH	0.1954	20.9867	20.9868
VAR(2)-fGARCH	0.1251	11.8767	13.8901
In-sample			
VAR(6)-GARCH	0.5954	22.9856	22.9888
VAR(6)-EWMA	0.5792	22.3034	22.3075
VAR(6)-GO-GARCH	0.4551	21.9932	21.9984
VAR(6)-fGARCH	0.43454	15.899	20.5455
Out-sample			
VAR(6)-GARCH	0.2991	23.8768	23.8798
VAR(6)-EWMA	0.2244	22.6741	22.6777
VAR(6)-GO-GARCH	0.2131	20.9957	20.9991
VAR(6)-fGARCH	0.1653	11.9934	13.9789

Conclusion

Tax revenue challenges as been come critical issues due to a significant number of financial crises that smash the global economy. Recently in Nigeria, there are extensive, unusual, and persistent financial crises (economic recession) and the falling price of crude oil in the international market particularly the state with a low and poor source of revenue and other factors.

In the real essence, the study on modeling and forecasting future tax revenue challenges good planning to withstand such economic crises. Modelling multivariate financial (tax revenue) parameters is vital in analyzing future economic indices. However, multivariate financial time series were characterized by variance volatility

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